Gains from Trade with Variable Trade Elasticities*

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Abstract

We show how to measure welfare gains from trade in static models where we make minimal assumptions on household preferences and production technologies. In many commonly studied environments, welfare gains depend on a crucial parameter: the elasticity of trade with respect to trade costs. Strong assumptions are required to ensure that this elasticity is a constant, which allows for welfare gains to be measured easily. Relaxing these assumptions, we prove two main results. First, the trade elasticity is no longer a constant, but we can characterize the variable trade elasticity as a function of preferences, production technologies and expenditure shares. Second, this variable trade elasticity is still sufficient to measure welfare, even in this more complex environment. We show how to apply our results to a multi-sector model with non-homotheticities, and show that trade elasticities vary substantially both across countries and levels of trade costs. Variable trade elasticity models have importantly different predictions for gains from trade than do constant trade elasticity models.

Keywords: Welfare gains from trade, variable trade elasticity, trade composition
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1 Introduction

Measuring the increase in welfare due to the expansion of international trade has been a central question in economics for decades. Answering this question is crucial for understanding the role of trade in recent growth episodes, and for predicting the effects of an expansion or contraction of trade. Arkolakis, Costinot and Rodríguez-Clare (2012, referred to hereafter as ACR) demonstrate that, for a large class of models, the answer to this question depends crucially on the trade elasticity, which is a parameter derived from a traditional gravity model (see Anderson and Wincoop, 2004; Baier and Bergstrand, 2009). Even though different models imply different structural interpretations of this parameter, in each of these models the gains from a marginal change in trade are pinned down by that single number.[1] However, not all trade models imply a constant trade elasticity. A growing body of the trade literature considers models that make use of technologies or preferences (such as non-homotheticities) that imply a variable (rather than constant) trade elasticity. The goal of these models is to match aspects of the data, such as how trade patterns vary with country characteristics (see, for example, Markusen, 1986; Fieler, 2011; Caron, Fally and Markusen, 2014; and Simonovska, 2015). The benefits of these richer environments come at the cost of analytical tractability. In turn, this lack of tractability has been a challenge for studying the welfare gains from trade in these environments.

In this paper, we show how to measure welfare gains from trade in models that exhibit a variable trade elasticity. Even in this more complex environment, the essential logic of the ACR result still holds: the crucial determinant of the welfare gains from trade is the trade elasticity. Unlike in the ACR environments, the elasticity may vary both across countries and with the level of trade costs. We show how to solve for the trade elasticity as a function of preferences, production technology, and the sectoral composition of expenditures. Finally, we provide results characterizing how the trade elasticity varies with trade costs. This methodology allows us to make direct comparisons between the welfare implications of models with a variable trade elasticity and those with a constant trade elasticity.

We then demonstrate how to apply these results in the context of Caron, Fally and Markusen (2014, henceforth CFM), which consists of a model with 50 sectors, and preferences that exhibit differing income elasticities in each sector. Production follows Eaton and Kortum (2002), but with different distributional parameters in each sector. This model does not generate closed form solutions for aggregate variables, and this lack of analytical tractability is a challenge for measuring gains from trade. Nonetheless, we show that our methodology can be applied easily to compute trade elasticities for each country in closed form. We find that country-level aggregate trade elasticities vary considerably across the 118 countries in the sample: the average trade elasticity is -7.5, which is the average trade elasticity in Anderson and Wincoop (2004), but the estimates range from -4.42 (Peru) to -11.37 (Luxembourg). We contrast this variable trade elasticity model with what one would get by naïvely applying the standard ACR formula to the same data. For the United States, the increase in real income from autarky to observed trade is 1.71%, almost doubling the 0.99% that would be implied by the ACR formula. For China, that same exercise yields gains of 2.73% and 1.42%, respectively. These differences are due to two types of variation: the variable trade elasticities

[1] For example, in Krugman (1980) it is related to a preference parameter, while in Eaton and Kortum (2002) it is equal to the distributional parameter governing comparative advantage across countries. In Chaney (2008), it is the tail parameter of the firm productivity distribution.

differ across countries (that is, it is not equal to -7.5 in each country) and across levels of trade costs (we find trade elasticities are closed to zero when countries are near autarky).

In this framework, aggregate trade elasticities (and, hence, gains from trade) vary systematically with the patterns of trade and production by sector. Countries that are less open to trade have trade elasticities that are closer to zero. The correlation between countries' import penetration and their trade elasticity is -0.602. This is due to how the composition of imports changes with trade costs. Sectors vary in their sectoral trade elasticities, so that imports in some sectors exhibit greater responsiveness to changes in trade costs than in others. When trade costs increase, imports fall by more in the more elastic sectors. As the country approaches autarky, all imports belong to the least elastic sector, and therefore the country's aggregate trade elasticity equals the elasticity in that most inelastic sector sector. This implies that the trade elasticity is closer to zero near autarky than at observed levels of trade. Correspondingly, the marginal gains to welfare are larger near autarky.

The second key characteristic is how much import penetration varies across sectors. When trade costs increase, imports decline and prices rise in every sector. However, prices increase relatively more in the most-imported sectors. This change in relative prices causes reallocation of the share of expenditure away from the most-imported sectors and toward the least-imported. Therefore, as more is spent in less-imported sectors, this reallocation causes aggregate imports fall, and pushes the trade elasticity away from zero. The magnitude of this effect depends on the variation in import penetration across sectors: greater variation causes more reallocation of expenditure. Therefore, countries with greater sectoral variation in import penetration realize smaller gains from trade.

Our results represent an extension of the ACR welfare gains formula to variable trade elasticity environments. Since our formula is a generalized version of their formula, the two formulae predict the same welfare gains from trade in environments with constant trade elasticity. Besides relaxing the assumption of a constant trade elasticity, the remaining assumptions in our model are the same as in ACR. The only assumptions we make with regard to the preference structure is that households have a strictly increasing, strictly concave and twice continuously differentiable utility function. This allows us to measure gains from trade in environments that are not normally analytically tractable.

As in ACR, we assume that aggregate profits are proportional wages. This assumption is important in that it does not allow us to consider environments that have pro-competitive effects of a trade liberalization, which is an important ongoing area of research. The closest paper to our analysis that considers pro-competitive effects from trade is Arkolakis et al. (forthcoming), which studies the welfare gains from trade in a model where markups change due to changes in trade costs. They find that the welfare elasticity is the reciprocal of the trade elasticity times a term that takes into account the average elasticity of markups with respect to firm productivity. Other papers that also consider pro-competitive environments include Holmes, Hsu and Lee (2014), Feenstra (2018), and Edmond, Midrigan and Xu (2015).

Other important extensions of the ACR framework that are close to our analysis are Costinot and Rodríguez-Clare (2014) and Ossa (2015), who build on multi-sector environments, where different sectors are aggregated using a Cobb-Douglas specification, and each sector has a (different) constant elasticity of substitution. They show that the trade elasticity that is relevant for the welfare gains from trade is a weighted average of the different sectoral trade elasticities. The Cobb-Douglas structure they impose implies that the trade elasticity is constant. By comparison, we do not impose neither a Cobb-Douglas specification of preferences nor a constant trade elasticity at the sector level.

In our application, we find a quantitatively important role for the reallocation effect resulting from expenditure shares changing with trade costs. Melitz and Redding (2014, 2015) also consider deviations from constant elasticity trade models and their implications for gains from trade. Melitz and Redding (2014) constructs an example with sequential production to show that gains from trade may be unboundedly high, even though gains from trade may be small at the margin. Melitz and Redding (2015) shows how deviations from the Pareto distribution in the Melitz (2003) model generate variable trade elasticity, and thus the ACR formula does not apply in that environment.

Our paper is closely related to the many papers that investigate gains from trade in models with sector-level heterogeneity, including Broda and Weinstein (2006), Ardelean and Lugoskyy (2010), Feenstra and Weinstein (2017) and Blonigen and Soderbery (2010). The strength of our paper is to nest many different environments and demonstrate how to directly relate measured trade elasticities to changes in welfare. Our paper is also related to Allen, Arkolakis and Takahashi (2014), who enrich a traditional trade model with an economic geography model and compute the (constant) trade elasticity in it. Since we are focussed on variable trade elasticity models, we view our paper as complementary to it. Lastly, recent work by Adao, Costinot and Donaldson (2017) provides a general framework to map non-parametric estimates of trade responses into gains from trade without having to specify many of the features of the underlying model. Our paper takes the opposite approach, starting with a model and providing an easy means of comparing implied welfare gains across models.

Our paper also contributes to recent theoretical literature that escapes the use of constant elasticity of substitution utility functions. For instance, Zhelobodko et al. (2012) study the role that non-constant elasticity of substitution models play in generating pro-competitive effects. Mrázová and Neary (2017) introduce the concept of a "demand manifold" and show that any well-behaved demand function can be represented by it. Finally, Dhingra and Morrow (forthcoming) study the allocational efficiency of markets with a high degree of productivity differences. Different from ours, this literature typically builds closed-economy models where some degree of monopolistic competition is present, which generates very interesting pro-competitive effects.

Though this paper studies static models, there is a growing body of research in international trade that is built around the variability of the trade elasticity over time. For instance, Ruhl (2008) builds a model that can account for both the low trade elasticity found in the international real business cycle literature and the high trade elasticity required to account for growth in trade after a trade agreement. Baier, Bergstrand and Feng (2014) document that the trade elasticity is variable over time, with the intensive margin responding quicker and having a larger effect than the extensive margin. Alessandria and Choi (2014*a*) show that the recent growth of U.S. manufacturing can be accounted for in models with dynamic export decision. Finally, Alessandria and Choi (2014*b*), Alessandria, Choi and Ruhl (2014) and Brooks and Pujolas (2017) show that the gains from trade that arise after a trade liberalization are larger when dynamics are taken into account.

2 Environment

We consider a two country model where households consume, inelastically supply labor and own firms, and where final good producers purchase intermediate goods from the domestic country and from abroad. The assumption on two countries is for clarity of exposition only; we show how to extend the results to more countries in Appendix A. We take the perspective that country 0 is the country of analysis, which we sometimes refer to as the "domestic" country, and country 1 is the rest of the world.

2.1 Household's problem

The household in country 0 has an additively separable utility function and solves the following problem:

$$\begin{aligned} &\max_{\{c(i)\}\geq 0} \int_{i\in\Omega} u_i(c(i))di \\ &\text{subject to: } L_0 \int_{i\in\Omega} p(i)c(i)di = I_0 \equiv w_0 L_0 + \Pi_0, \end{aligned} \tag{1}$$

where $c(i)$ is consumption of the output of sector i and $p(i)$ is the price level in sector i. There is a set Ω of sectors that produce using domestic and foreign intermediates. The wage in country 0 is w_0, which we make the numéraire. The measure of identical households in country 0 is L_0, and each household supplies one unit of labor inelastically. Households own the firms, and receive a lump sum Π_0 of all profits generated in country 0.

We do not specify a utility function in problem (1), and we assume only that u_i be strictly increasing, strictly concave and twice differentiable for each sector i. These preferences may be non-homothetic. The non-negativity constraint on consumption expenditure implies that there may be an extensive margin in the consumption of different sectors. The assumption on additively separable utility function is made for clarity of exposition in the body of this paper, and we show how to extend the results to the non-separable case in Appendix B. Finally, in Appendix C, we also show how our results apply in cases where household decisions are represented by demand systems rather than utility functions (for instance, the Almost Ideal Demand System).

2.2 Production

Within each sector there is a set of foreign and domestic firms that sell their output to country 0. Final output in each sector comes from aggregating the output from these firms according to an aggregator function F_i. The price of the final good incorporates trade costs from imports, as well as the prices of all intermediates.

The price level, $p(i)$, is the solution to the problem:

$$\begin{aligned} p(i) = &\min_{\{x_{0n}(i,j)\}\geq 0} \sum_{n=0}^{1} \int_{j\in\Upsilon_n(i)} \tau_{0n} q_n(i,j) x_{0n}(i,j)dj \\ &st: 1 = F_i\left(\{\{x_{0n}(i,j)\}_{j\in\Upsilon_n(i)}\}_n\right), \end{aligned} \tag{2}$$

where $p(i)$ is the price of good i; $x_{0n}(i,j)$ is a the quantity of variety j used in the production of good i, $q_n(i,j)$ is the price of this good, and $\Upsilon_n(i)$ is the set of varieties available from country n in sector i. The iceberg trade costs τ_{0n} are the number of units that have to be produced in country n in order for 1 unit to be sold to country 0. Hence, these costs are not redistributed back to consumers. We consider trade costs that are equal in all sectors.

We make three assumptions about this problem. First, F_i is constant returns to scale. Second, the prices of intermediate goods $q_n(i,j)$ are linear in country n wages, and independent of trade costs and wages in other countries. Third, aggregate profits from the production sector in country 0, Π_0, are proportional to the wage in country 0.

This third assumption is most clearly satisfied when each sector exhibits perfect competition. There are other environments that also satisfy this assumption, such as when markups are constant and equal across sectors. For clarity of exposition, we concentrate on the perfect competition case in the remainder of the paper.

Last, labor markets in each country clear by an equilibrium wage. Importantly, wages may differ across countries so that there are non-trivial general equilibrium effects from changes in trade costs.

3 Trade and Welfare Elasticities

In this section we show how to compute the welfare gains from trade in the model described in the previous section. We proceed in two steps. First, we solve for the trade elasticity function in this environment. This function depends on preferences, production technologies, and expenditures. In general, this trade elasticity is not a constant, and may vary by country and with trade costs. For this reason, we refer to it as a "variable trade elasticity" in contrast to the "constant trade elasticity" models studied in ACR. Second, we link the variable trade elasticity to gains from trade. The second step requires a formal proof, as it is not implied by the ACR result.

3.1 Characterization of Variable Trade Elasticity Function

We denote expenditure by country 0 on goods produced in country n in sector i as:

$$X_{0n}(i) = \int_{\Upsilon_n(i)} \tau_{0n} q_n(i,j) x_{0n}(i,j) dj. \tag{3}$$

Total expenditure on country n goods by country 0 is simply the sum of these sectoral expenditures:

$$X_{0n} = \int_{\Omega} X_{0n}(i) di. \tag{4}$$

The aggregate trade elasticity, ε_T, is a partial elasticity defined as:

$$\varepsilon_T = \frac{\partial \log(X_{01}/X_{00})}{\partial \log(\tau_{01})}. \tag{5}$$

This partial elasticity is the change in imports relative to domestic expenditure due to a change in the trade costs holding wages and the measure of goods that can be produced in each country fixed.

Only in specific environments is the trade elasticity constant, and the assumptions made in the previous section do not imply that this elasticity is constant.[2] Instead, we now solve for the trade elasticity as a function of preferences, technologies and expenditures. To do so, it is useful to characterize two terms that may vary at the sector level. The first is a sector-level partial trade elasticity that gives the change in the mix of domestic and foreign expenditure share in response to

[2] For this elasticity to be constant, preferences need to exhibit constant elasticity of substitution, and the distribution of productivity needs to be of a particular type: Fréchet in the case of the Eaton and Kortum (2002) model and Pareto in the case of the Melitz (2003) model. These are, precisely, the environments that ACR consider.

a change in trade costs:

$$\rho(i) = \frac{\partial \log(X_{01}(i)/X_{00}(i))}{\partial \log(\tau_{01})}.$$

As with the aggregate trade elasticity ε_T, these sectoral partial trade elasticities hold wages and the measure of goods fixed. Likewise, these sectoral trade elasticities need not be constant. We can characterize $\rho(i)$ as a function of the production technology F_i as follows in the case of perfect competition where there is only one competitively produced domestic product, $y_d(i)$, and one import, $y_x(i)$, in each sector. For

$$\Omega_{xx}(i) \equiv \frac{y_x(i)}{y_d(i)} \frac{F_{xxi}(i)}{F_(xi)},$$

$$\Omega_{dx}(i) \equiv \frac{y_x(i)}{y_d(i)} \frac{F_{dxi}(i)}{F_(di)},$$

then

$$\rho(i) = 1 - \frac{1}{\Omega_{xx}(i) - \Omega_{dx}(i)}, \tag{6}$$

If F_i is a constant elasticity of substitution aggregator, then this is a constant. But in general it is not.

Another key element of the aggregate trade elasticity is how households substitute their expenditure across sectors as trade costs change. This is summarized by a preference term for each sector that depends on the curvature of the household's utility function. These terms are defined in each sector as:

$$\kappa(i) = \begin{cases} 0 & \text{if } c(i) = 0 \\ \frac{u_i'(c(i))}{c(i)u_i''(c(i))} & \text{if } c(i) > 0 \end{cases}. \tag{7}$$

These terms, together with expenditure shares by sector and origin, are sufficient to compute the trade elasticity. We show this result in Proposition 1.

Proposition 1 *Whenever $X_{01} > 0$:*

$$\varepsilon_T = \int_\Omega \rho(i) \frac{\omega_{01}(i)\omega_{00}(i)}{\omega_0(i)} di - \int_\Omega (1 + \kappa(i))\, \omega_{01}(i) \left[\frac{\omega_{00}(i)}{\omega_0(i)} - \frac{\int_\Omega \kappa(j)\omega_{00}(j)dj}{\int_\Omega \kappa(k)\omega_0(k)dk} \right] di, \tag{8}$$

where

$$\omega_{0j}(i) = \frac{X_{0j}(i)}{X_{0j}},$$

$$\omega_0(i) = \frac{X_{00}(i) + X_{01}(i)}{X_{00} + X_{01}}.$$

The proof of Proposition 1 is in Appendix D. The formula for the trade elasticity, equation (8), describes how import intensity changes when there is a change in trade cost. Aggregate imports depend on two things: the fraction of imports in each sector, and the distribution of expenditure across sectors. Aggregate imports could decrease either because imports within each sector falls, or because households shift expenditure from sectors with high import content to sectors with low import content. The two terms in equation (8) correspond to each of these effects. The first term, which depends on sectoral trade elasticities, shows how changes in trade costs affect import content in each sector, and how those effects are aggregated to affect total imports. The second term, which depends on household preferences, shows how household reallocation of expenditure across sectors in response to a change in trade costs affects total imports. If households shift their expenditure toward more heavily imported sectors, that causes aggregate imports to increase, while if they shift toward less imported sectors, aggregate imports fall. When preferences are Cobb-Douglas, which corresponds to $\kappa(i) = -1$ for all i, this reallocation effect is absent. Likewise, when import intensity is equal in all sectors, which corresponds to $\omega_{01}(i) = \omega_{00}(i) = \omega_0(i)$ for all i, it is clear that the second term is equal to zero and the reallocation effect is absent.

In Appendix E, we provide the values of κ_i and ρ_i that correspond to a number of example environments. These examples demonstrate that $\rho(i)$ and $\kappa(i)$ need not be invariant either across countries or across value of the trade cost τ_{01}.

3.2 Welfare Elasticity

In this section we show how to use the trade elasticity given by equation (8) to measure the gains from a small reduction in trade costs. We derive a version of the ACR result in this environment with variable trade elasticities.

To begin, we define the domestic and foreign expenditure shares as:

$$\lambda_{0j} = \frac{X_{0j}}{X_{00} + X_{01}}. \tag{9}$$

Our measure of the welfare gains from trade is compensating variation, defined as the increase in income needed to make the household indifferent to a marginal increase in trade costs. Formally, the expenditure function is:

$$\begin{aligned} e(\bar{u}, \tau_{01}) = \quad & \min \int_{i\in\Omega} p(i)c(i)di \\ \text{subject to:} \quad & \bar{u} = \int_{i\in\Omega} u_i(c(i))di. \end{aligned} \tag{10}$$

The ratio of $e(\bar{u}, \tau_{01})$ and any $e(\bar{u}, \tau'_{01})$ is the proportional loss of real income due to a change in trade costs.[3] We then define the "welfare elasticity" as how welfare changes with a small change in the domestic expenditure share, λ_{00}, when both are changing due to a change in trade costs. Namely,

$$\varepsilon_W \equiv -\frac{\frac{\partial \log(e(\bar{u},\tau_{01})}{\partial \log(\tau_{01})}}{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}. \tag{11}$$

In Proposition 2 we show that the two elasticities, the trade elasticity in equation (8), and the welfare elasticity in equation (11), are related.

Proposition 2 $\forall \lambda_{00} \in (0, 1)$,

$$\varepsilon_W = \frac{1}{\varepsilon_T}.$$

The proof of Proposition 2 is in Appendix F. Taken together, Propositions 1 and 2 provide a means to first solve for the trade elasticity, and then to use the trade elasticity to measure the welfare gains from trade.

[3] In models with a perfect price index, like those considered in ACR, the ratio of expenditure functions is equal to the ratio of price indexes. Therefore, our measure of welfare coincides with the one used in that paper.

4 Large Changes in Trade Costs

The results developed in Section 3 are useful for measuring the gains from small changes in trade costs by showing how to solve for the trade and welfare elasticities at observed levels of trade. However, the environments considered in this paper exhibit trade elasticities that change as the economy moves away from observed levels of trade. In this section we develop results to characterize this variation.

Variation in trade costs matters when changes in trade costs are nontrivial. At the extreme, consider the welfare effects of moving from an initial level of trade costs, τ_{01}, to a level of trade costs sufficient to put the economy into autarky, τ_{01}^{AUT}. Integrating over the marginal gains gives the total effect on the logarithm of welfare as:

$$\int_{\log(\tau_{01})}^{\log(\tau_{01}^{AUT})} \varepsilon_W(\tau) \frac{\partial \log(\lambda_{00})}{\partial \log(\tau)} d\log(\tau) = \int_{\log(\tau_{01})}^{\log(\tau_{01}^{AUT})} \frac{1}{\varepsilon_T(\tau)} \frac{\partial \log(\lambda_{00})}{\partial \log(\tau)} d\log(\tau). \tag{12}$$

Notice that this can be integrated by parts to yield:

$$\int_{\log(\tau_{01})}^{\log(\tau_{01}^{AUT})} \varepsilon_W(\tau) \frac{\partial \log(\lambda_{00})}{\partial \log(\tau)} d\log(\tau) = \frac{\log(\lambda_{00}(\tau_{01}))}{\varepsilon_T(\tau_{01})} - \int_{\log(\tau_{01})}^{\log(\tau_{01}^{AUT})} \frac{\log(\lambda_{00}(\tau))}{\varepsilon_T(\tau)^2} \frac{\partial \varepsilon_T(\tau)}{\partial \log(\tau)} d\log(\tau). \tag{13}$$

The first term of equation (13) is exactly the formula for the gains from trade in ACR, while the second term depends on how the the trade elasticity changes as trade costs change. The sign of the second term precisely depends on how the trade elasticity is changing. In particular, if the trade elasticity function is closer to zero near autarky, then the marginal increases in import intensity near autarky are more valuable than those near observed levels of trade. Hence, if one were to use the ACR formula to measure gains from trade with the trade elasticity evaluated near observed levels of trade, one would underestimate the welfare effects of moving to autarky. The opposite is true if the trade elasticity is further from zero when the economy is closer to autarky. This is formalized in the following proposition.

Proposition 3 *If the variable trade elasticity function is monotonically increasing (decreasing), then the welfare gains of moving from autarky to trade costs equal to* τ_{01} *are higher (lower) than:*

$$\lambda_{00}(\tau_{01})^{\frac{1}{\varepsilon_T(\tau_{01})}} - 1. \tag{14}$$

The proof of Proposition 3 is in Appendix G.

Since Proposition 3 demonstrates the importance of how ε_T changes over trade costs, we next provide some results to characterize changes in ε_T. In particular, as the formula in Proposition 1 makes clear, there are three reasons that ε_T might change with trade costs τ_{01}. First, the sectoral elasticities $\rho(i)$ may change with trade costs. Second, the preference terms $\kappa(i)$ may change with trade costs. Finally, the composition of expenditure across sectors may change with trade costs. In general, the total change in ε_T for a change in τ_{01} depends on all three of these effects simultaneously.

4.1 Changes in the Production Term, $\rho(i)$

Here we continue with the case with perfect competition and a single input from each market as discussed in equation (6). Following the notation before, the elasticity of substitution is:

$$\xi(i) = \frac{1}{\Omega_{xx}(i) - \Omega_{dx}(i)}. \tag{15}$$

Then it is straightforward to show that $\rho(i) = 1-\xi(i)$. Therefore, changes in $\rho(i)$ can be characterized by measuring changes in $\xi(i)$. Because $\xi(i)$ is itself an elasticity, the effect of changes in τ depends on third derivative terms given by:

$$\Omega_{xxx}(i) = \frac{y_x(i)}{y_d(i)} \frac{F_{xxx}(i)}{F_{xx}(i)}, \qquad \Omega_{dxx}(i) = \frac{y_x(i)}{y_d(i)} \frac{F_{dxx}(i)}{F_{dx}(i)}. \tag{16}$$

Then as τ varies, the change in the elasticity of substitution is given by:

$$\frac{\partial \log(\xi(i))}{\partial \log(\tau)} = \frac{1 - \Omega_{xx}(i) - \Omega_{dx}(i)}{\Omega_{dx}(i) - \Omega_{xx}(i)} - \frac{\Omega_{xx}(i)\Omega_{xxx}(i) - \Omega_{dx}(i)\Omega_{dxx}(i)}{(\Omega_{dx}(i) - \Omega_{xx}(i))^2}. \tag{17}$$

The third derivative terms control how the substitutability between domestic and foreign inputs varies as their relative utilization changes. That is, if imports are less substitutable with domestic inputs when imports are high relative to when they are low, that implies that $\xi(i)$ is an increasing function of τ. Then, since $\rho(i) = 1 - \xi(i)$, this implies that $\rho(i)$ is a decreasing function of τ. In general, how $\rho(i)$ changes in τ is given by the formula:

$$\frac{d \log(\rho(i))}{d \log(\tau)} = \frac{\xi(i)^2(1 - \Omega_{xx}(i) - \Omega_{dx}(i)) + \xi(i)^3(\Omega_{xxx}(i) - \Omega_{dxx}(i))}{1 - \xi(i)}. \tag{18}$$

4.2 Changes in the Preference Term, $\kappa(i)$

The preference terms $\kappa(i)$ describe how the composition of expenditure changes with trade costs. As discussed in Section 3, $\kappa(i)$ is determined by the curvature of the utility function u_i. As trade costs vary, $\kappa(i)$ varies if the curvature of u_i varies with consumption in sector i, $c(i)$. In particular, we can write this as:

$$\frac{\partial \log(\kappa(i))}{\partial \log(\tau)} = \frac{\partial \log(c(i))}{\partial \log(\tau)} \left[\frac{c_i u_i''}{u_i'} - \frac{c_i u_i'''}{u_i''} - 1 \right]. \tag{19}$$

That is, $\kappa(i)$ changes as consumption moves along the function u_i. Therefore, the effect on $\kappa(i)$ from a change in τ depends on how much consumption c_i changes, and how $\kappa(i)$ is changing in c_i.

As in the derivation of the trade elasticity ε_T in Section 3, we can solve for the changes in consumption as a function of expenditures and $\kappa(i)$, and then substitute that into this expression:

$$\frac{\partial \log(\kappa(i))}{\partial \log(\tau)} = \left[\kappa(i)(1 - \lambda_{00}(i)) - \kappa(i) \frac{\sum_j (1 + \kappa(j)) X_{01}(j)}{\sum_l \kappa(l) X_0(l)} \right] \left[\frac{c_i u_i''}{u_i'} - \frac{c_i u_i'''}{u_i''} - 1 \right]. \tag{20}$$

The sign of the first term, which is how consumption c_i is changing, depends on how import penetration in sector i compares to import penetration in other sectors. In general, because increases in trade costs cause an increase in prices in each sector, we would expect quantities purchased to go down. However, sectoral prices change in proportion to import penetration in each sector. Therefore, sectors with very high import penetration have much larger increases in prices than sectors with low import penetration. This change in relative prices may cause the quantity of consumption to increase in the least-imported sectors, which would make the first term positive. In any other case, the first term is negative.

The sign of the second term depends on the shape of the utility function. If the utility function u_i has the form $u_i(c_i) = c_i^{\phi}$, then the second term is zero. Otherwise, the sign of this term depends on whether the magnitude of the third derivative of the utility function.

4.3 Special Case: Changes in Composition

To focus on the issue of how changes in the composition of imports and domestic consumption affect the trade elasticity, we consider the special case where the preference terms $\kappa(i)$ and production elasticities $\rho(i)$ are all constant. Because they are not equal to one another, the trade elasticity ε_T is variable as the composition of expenditure changes with trade costs. However, as the country approaches autarky, the trade elasticity converges to the sectoral elasticity of the least elastic sector. We prove this result in Proposition 4 next.

Proposition 4 *Let* $\rho^{MIN} = \min\{\rho(i)\}$. *Then:*

$$\lim_{\tau_{01}\to\infty} \varepsilon_T = -\rho^{MIN}.$$

See Appendix H for the proof.

5 Application to a Quantitative Model

We now provide an example on how our results apply to a framework with a variable trade elasticity. We choose to work with the framework of Caron, Fally and Markusen (2014), which we hereafter refer to as CFM, because it is consistent with many patterns of trade that are not matched by constant trade elasticity models, and cannot be solved in closed form.[4]

Households have preferences described by constant relative income elasticity (CRIE) utility functions given by:

$$u_i(c(i)) = \frac{\alpha(i)\sigma(i)c(i)^{\frac{\sigma(i)-1}{\sigma(i)}}}{\sigma(i)-1} \tag{21}$$

Here, $\alpha(i)$ is a sector-specific weighting parameter, and $\sigma(i)$ controls relative income elasticity. Applying the definition of $\kappa(i)$ to these preferences, this model implies $\kappa(i) = -\sigma(i)$ for each i. Moreover, production follows Eaton and Kortum (2002) with a comparative advantage parameter in the Frechet distribution given by $\theta(i)$ in each sector, and explicitly allows for the possibility that these parameters may vary across sectors. This implies that the sector-level trade elasticities are $\rho(i) = -\theta(i)$.

Applying Proposition 1, the trade elasticity in this model is given by

$$\varepsilon_T = -\sum_{i=1}^{50}\theta(i)\frac{\frac{X_{01}(i)}{X_{01}}\frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}} - \sum_{i=1}^{50}(1-\sigma(i))\frac{X_{01}(i)}{X_{01}}\left(\frac{\frac{X_{00}(j)}{X_{00}}}{\frac{X_0(j)}{I_0}} - \left[\frac{\sum_j \sigma(j)\frac{X_{00}(j)}{X_{00}}}{\sum_j \sigma(j)\frac{X_0(j)}{I_0}}\right]\right), \tag{22}$$

[4] CFM presents several specifications. Here we are using the "theta-driven" model, which has sector heterogeneity in comparative advantage and a single factor of production.

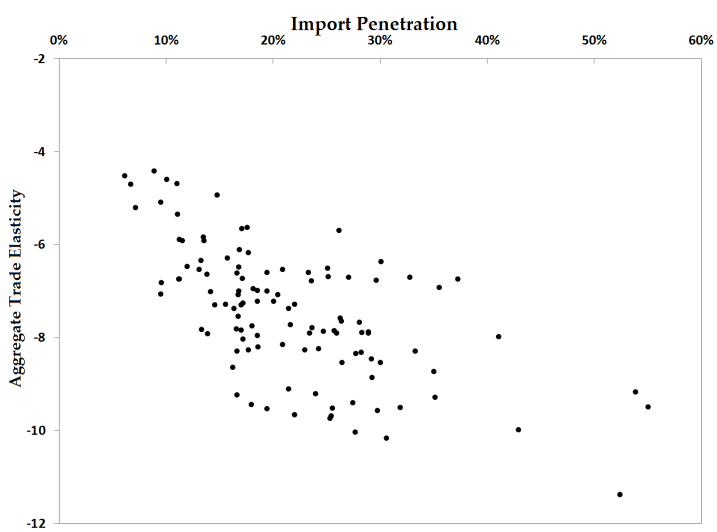
where the summations are over the 50 sectors in our dataset. The parameters $\theta(i)$ and $\sigma(i)$ are estimated in CFM, so given data on expenditures by sector, the trade elasticity can then be computed for each country using this formula. As in CFM, we use expenditure share data from the GTAP dataset of Narayanan, Aguiar and McDougall (2012), which includes production and trade data by sector in a large number of countries.[5]

One challenge with applying the elasticity estimates from CFM is that their empirical strategy only identifies the set of $\theta(i)$ and $\sigma(i)$ terms up to multiplicative constants for each group of elasticity estimates. Therefore, we are left with two degrees of freedom in choosing the averages of these vectors. We choose two targets. First, we match the average value of $\theta(i)$ across sectors to 4 based on Simonovska and Waugh (2014), which identifies the shape parameter in the Eaton and Kortum (2002) model correcting for finite sample bias.[6] Second, using all data and the estimates of $\theta(i)$, we choose the average of $\kappa(i)$ to set the average country-level aggregate trade elasticity to -7.5, which is the midpoint of the range of trade elasticity estimates from Anderson and Wincoop (2004).

5.1 Measuring the Trade and Welfare Elasticities

The trade elasticity is computed using the formula in equation (22). In the second column of Table 1 we report the elasticity values. In the first column, we include import penetration ratios equal to $1-\lambda_{00}$. Our analysis shows that aggregate trade elasticities range widely from -4.42 (Peru) to -11.37 (Luxembourg), with a standard deviation of 1.41 and average of -7.5. By Proposition 2, the welfare elasticity is just the reciprocal of these trade elasticities. Therefore the welfare elasticities range from -0.09 to -0.23, meaning that a marginal change in import penetration has a 2.57 greater effect on real income in Peru than in Luxembourg. There is also a quite strong correlation (-0.61) between import penetration and the trade elasticity. Figure 1 illustrates this relationship in a scatterplot.

Figure 1: Trade Elasticity by Import Penetration



This result is a direct consequence of Proposition 4: as each country approaches autarky, country-

[5] From GTAP 8, we make use of data from 118 countries (see Table 1) and following CFM, we remove seven sectors composed of raw materials or those that are not traded, leaving fifty sectors spanning agriculture, manufacturing and services. We use data on country-sector imports, and country-sector final expenditure.

[6] Our choice of 4 is close to the midpoint of the range they estimate, which is 2.79 to 4.46.

Table 1: Trade Elasticity and Decomposition

Country	Import Penetration	Trade Elasticity	Production Term (Θ)	Preference Term (Σ)	Country	Import Penetration	Trade Elasticity	Production Term (Θ)	Preference Term (Σ)
Albania	28.26%	-7.89	-2.32	-5.57	Latvia	26.27%	-7.58	-2.06	-5.52
Algeria	16.70%	-7.54	-2.00	-5.54	Lithuania	25.90%	-7.90	-1.96	-5.94
Argentina	11.19%	-6.73	-2.22	-4.51	Luxembourg	52.37%	-11.37	-1.96	-9.41
Armenia	16.59%	-9.23	-1.72	-7.52	Madagascar	17.18%	-8.02	-1.72	-6.30
Australia	9.52%	-6.81	-2.25	-4.56	Malawi	19.42%	-9.53	-1.40	-8.13
Austria	23.41%	-7.90	-2.12	-5.78	Malaysia	29.60%	-6.76	-2.11	-4.65
Azerbaijan	25.54%	-9.52	-2.01	-7.51	Malta	53.83%	-9.16	-1.89	-7.27
Bahrain	27.63%	-10.04	-1.43	-8.61	Mauritius	37.24%	-6.73	-2.19	-4.54
Bangladesh	13.82%	-6.64	-1.97	-4.67	Mexico	13.84%	-7.91	-1.85	-6.06
Belarus	18.59%	-8.20	-1.75	-6.45	Moldova	34.99%	-8.72	-1.91	-6.82
Belgium	31.88%	-9.50	-1.73	-7.77	Mongolia	30.56%	-10.17	-1.33	-8.83
Belize	26.13%	-5.69	-2.55	-3.15	Morocco	17.55%	-5.62	-2.44	-3.19
Benin	42.94%	-9.97	-0.87	-9.10	Mozambique	30.03%	-8.53	-1.66	-6.87
Bolivia	16.19%	-8.63	-1.84	-6.79	Namibia	24.30%	-8.91	-1.59	-7.31
Botswana	23.39%	-9.91	-1.42	-8.49	Nepal	16.98%	-7.84	-1.85	-5.99
Brazil	6.15%	-4.51	-2.77	-1.75	Netherlands	16.75%	-7.08	-2.43	-4.65
Bulgaria	24.70%	-7.87	-2.11	-5.76	New Zealand	11.24%	-5.88	-2.50	-3.38
Burkina Faso	17.15%	-7.26	-1.89	-5.36	Nicaragua	28.23%	-8.31	-1.84	-6.47
Cambodia	33.25%	-8.29	-1.40	-6.89	Nigeria	22.97%	-8.26	-1.75	-6.51
Cameroon	11.08%	-5.34	-2.62	-2.72	Norway	16.76%	-6.48	-2.49	-3.99
Canada	14.17%	-7.01	-2.25	-4.76	Oman	29.74%	-9.57	-1.65	-7.92
Chile	14.55%	-7.29	-2.13	-5.16	Pakistan	13.33%	-7.82	-1.93	-5.90
China	10.03%	-4.59	-2.51	-2.08	Panama	25.43%	-9.68	-1.31	-8.37
Colombia	9.47%	-7.06	-1.96	-5.10	Paraguay	21.41%	-9.10	-1.39	-7.71
Costa Rica	24.24%	-8.24	-1.78	-6.46	Peru	8.86%	-4.42	-2.70	-1.72
Cote d'Ivoire	17.13%	-6.72	-2.16	-4.56	Philippines	22.02%	-7.28	-1.93	-5.35
Croatia	19.44%	-7.00	-2.21	-4.79	Poland	16.62%	-6.60	-2.20	-4.40
Cyprus	28.89%	-7.88	-2.00	-5.87	Portugal	15.69%	-6.28	-2.39	-3.89
Czech Republic	23.30%	-6.60	-2.17	-4.43	Qatar	20.05%	-7.22	-2.06	-5.16
Denmark	23.54%	-6.77	-2.63	-4.15	Romania	16.77%	-6.99	-2.06	-4.93
Ecuador	16.32%	-7.37	-2.09	-5.28	Russia	11.93%	-6.47	-2.36	-4.11
Egypt	19.39%	-6.59	-2.31	-4.28	Rwanda	18.01%	-7.75	-1.86	-5.88
El Salvador	20.87%	-6.54	-2.22	-4.32	Saudi Arabia	30.10%	-6.36	-2.81	-3.55
Estonia	29.25%	-8.85	-1.73	-7.13	Senegal	28.05%	-7.66	-1.93	-5.73
Ethiopia	17.67%	-8.27	-1.54	-6.73	Singapore	35.51%	-6.91	-2.45	-4.47
Finland	17.03%	-5.66	-2.49	-3.17	Slovakia	23.61%	-7.78	-2.01	-5.77
France	11.52%	-5.91	-2.45	-3.47	Slovenia	26.37%	-7.64	-2.01	-5.63
Georgia	23.97%	-9.20	-1.45	-7.75	South Africa	11.21%	-5.52	-2.50	-3.02
Germany	16.83%	-6.10	-2.49	-3.62	South Korea	14.78%	-4.93	-2.63	-2.30
Ghana	21.61%	-7.72	-1.79	-5.93	Spain	13.47%	-5.83	-2.53	-3.30
Greece	18.50%	-6.98	-2.21	-4.77	Sri Lanka	20.43%	-7.07	-1.90	-5.16
Guatemala	20.86%	-8.15	-1.66	-6.49	Sweden	17.66%	-6.17	-2.47	-3.69
Guinea	27.74%	-8.34	-1.68	-6.65	Switzerland	25.30%	-9.73	-1.68	-8.05
Honduras	29.17%	-8.46	-1.44	-7.02	Taiwan	25.10%	-6.51	-2.17	-4.34
Hong Kong	26.44%	-8.53	-1.97	-6.57	Tanzania	21.99%	-9.66	-1.27	-8.39
Hungary	27.02%	-6.70	-2.24	-4.46	Thailand	25.12%	-6.69	-2.18	-4.51
Iceland	21.45%	-7.37	-2.22	-5.15	Togo	55.03%	-9.48	-1.07	-8.41
India	9.50%	-5.09	-2.63	-2.46	Tunisia	28.88%	-7.90	-1.84	-6.07
Indonesia	13.52%	-5.91	-2.43	-3.48	Turkey	13.09%	-6.53	-2.12	-4.40
Iran	18.11%	-6.94	-2.29	-4.65	UAE	35.09%	-9.28	-1.57	-7.70
Ireland	32.79%	-6.70	-2.80	-3.91	Uganda	16.61%	-8.29	-1.74	-6.55
Israel	18.54%	-7.21	-2.24	-4.97	Ukraine	18.51%	-7.95	-2.00	-5.94
Italy	11.03%	-4.68	-2.82	-1.86	United Kingdom	13.27%	-6.33	-2.52	-3.81
Japan	6.66%	-4.70	-2.70	-1.99	United States	7.16%	-5.20	-2.56	-2.64
Kazakhstan	17.02%	-7.29	-2.35	-4.95	Uruguay	16.55%	-7.81	-2.04	-5.77
Kenya	15.56%	-7.28	-1.93	-5.34	Venezuela	11.20%	-6.73	-2.17	-4.57
Kuwait	25.69%	-7.85	-2.31	-5.54	Vietnam	41.05%	-7.98	-1.57	-6.40
Kyrgyzstan	27.42%	-9.40	-1.37	-8.03	Zambia	13.50%	-9.44	-1.64	-7.80
Laos	17.98%	-9.44	-1.27	-8.16	Zimbabwe	38.07%	-7.81	-2.15	-5.66

level trade elasticities approach the sectoral trade elasticity that is closest to zero. In our parametrization, this limit is -2.21, as the industry with the lowest $\theta(i)$ is leather products. This result says that, near autarky, that sector accounts for nearly all imports and therefore its sector-level elasticity is equal to the aggregate trade elasticity. Here, the welfare elasticity near autarky is -0.45, which is more than twice the marginal effect of any country in the sample. This demonstrates that as countries approach autarky, their aggregate trade elasticities are closer to zero than they are at the observed levels of trade. Notice that the fact that the composition of expenditures can vary in this model is crucial for this result.

5.2 Decomposition of Trade Elasticity

Besides the negative relationship between import penetration and trade elasticity, another interesting feature that comes clearly from Figure 1 is that even among countries of similar import penetration levels, there exists considerable variation in trade elasticities. We provide a decomposition to understand this heterogeneity. Notice that the trade elasticity, equation (22), can be broken into two terms:

$$\Theta = -\sum_{i=1}^{50} \theta(i) \frac{\frac{X_{01}(i)}{X_{01}} \frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}},$$

$$\Sigma = \sum_{i=1}^{50} (\sigma(i) - 1) \frac{X_{01}(i)}{X_{01}} \left[\frac{\frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}} - \frac{\sum_j \sigma(j) \frac{X_{00}(j)}{X_{00}}}{\sum_k \sigma(k) \frac{X_0(k)}{I_0}} \right].$$

The Θ term includes only production terms, and the Σ term includes only preference terms. These are reported in columns 3 and 4 of Table 1.[7] The Σ term has much more variation across countries than the Θ term. Note that we can write the usual decomposition of variance as

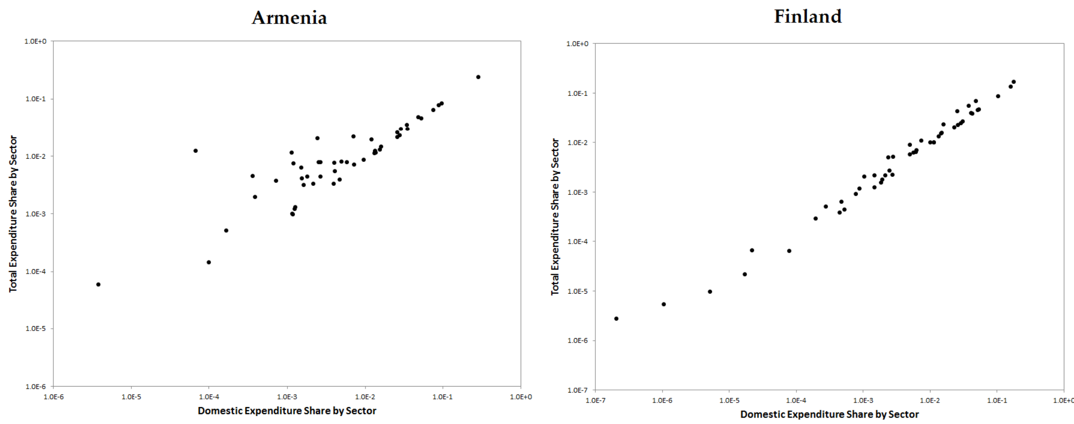
$$\varepsilon_T = \Theta + \Sigma \implies Var(\varepsilon_T) = Var(\Theta) + Var(\Sigma) + 2Cov(\Theta, \Sigma).$$

The variance of trade elasticities is 1.99, the variance of Θ is 0.17, the variance of Σ is 3.15, and the covariance of Θ and Σ is -0.66. This shows that the preference terms are the major determinants of cross-country variation in aggregate trade elasticities.

To understand why there is so much variation in Σ we now consider two countries with very similar import penetration, but very different trade elasticities: Finland and Armenia. As shown in Table 1, Finland has import penetration of 17.03% and a trade elasticity of -5.66, while Armenia has import penetration of 16.59% and a trade elasticity of -9.23. From the results of the decomposition on Table 1, we can again see that the difference in the Σ term is responsible for the large difference in trade elasticities, since Θ is smaller in Armenia than in Finland. This difference is due to the fact that Finland has much less variation in import penetration at the sector level than does Armenia. That is, while the aggregate import penetration of the countries is similar, Armenia has some industries with extremely high import penetration and others very low. This is illustrated in Figure 2 by plotting the domestic expenditure share of each sector against its total expenditure share. If import penetration was constant across sectors, this scatterplot would follow the 45 degree line. It is evident that this is much nearer to true in Finland than in Armenia.

[7] Notice that the weights appearing in the definition of Θ add to a number less than one whenever there is cross-sectoral variation in import penetration. Therefore, even though the average value of ρ is -4, all countries have a Θ greater than -4.

Figure 2: Comparing Domestic and Total Expenditure Shares, Armenia and Finland



The reason that the cross-sectoral variation in import penetration is so important for the aggregate trade elasticity is precisely because of the reallocation effect. A change in trade costs has a larger impact on prices in sectors with greater import penetration. This greater variation in import penetration increases in trade costs cause greater changes in relative prices across sectors and therefore greater reallocation. In particular, households shift spending away from more highly imported sectors to less imported sectors. This change in composition of expenditure reduces aggregate imports from a change in trade costs.

Consistent with this explanation, if import penetration was exactly equal in each sector, then it is immediate to verify that, for all i

$$\frac{X_{00}(i)}{X_{00}} = \frac{X_0(i)}{I_0} \implies \Sigma = 0.$$

This effect is absent in models where the composition of expenditure across sectors is fixed, such as when preferences are Cobb-Douglas. In our environment, that corresponds to the case where $\sigma(i) = 1$ for each i. Again, in that case it is easy to verify that $\Sigma = 0$ and the reallocation effect is absent.

5.3 Comparison to Autarky

Now we measure changes in real income moving from autarky to observed levels of trade and compare the increase in real income to the increase in income implied by constant trade elasticity models. In Section 4 we show that the main determinant of the increase in real income is the derivative of the trade elasticity with respect to trade costs. As Proposition 4 demonstrates, every country's trade elasticity near autarky is known, and is closer to zero than any country's trade elasticity at observed levels of trade. This suggests that variable trade elasticities matter for the welfare gains from trade. If we were to approximate the gains from trade using the ACR formula in this richer environment, we would systematically understate the gains from trade. In this exercise, we seek to quantify the difference between the actual gains from trade and what one would obtain from the ACR formula.

To measure the size of this effect we compute the real income needed to make each country indifferent between observed levels of trade and autarky. To solve for the equilibrium in autarky

Table 2: Gains from Trade: Autarky to Observed Trade

Country	Trade Elasticity	ACR Gains (constant)	ACR Gains (marginal)	True Gains	Country	Trade Elasticity	ACR Gains (constant)	ACR Gains (marginal)	True Gains
Albania	-7.89	4.53%	4.30%	5.26%	Latvia	-7.58	4.15%	4.10%	4.87%
Algeria	-7.54	2.47%	2.45%	3.05%	Lithuania	-7.90	4.08%	3.87%	4.67%
Argentina	-6.73	1.59%	1.78%	2.20%	Luxembourg	-11.37	10.40%	6.74%	8.62%
Armenia	-9.23	2.45%	1.98%	2.51%	Madagascar	-8.02	2.54%	2.38%	3.44%
Australia	-6.81	1.34%	1.48%	1.83%	Malawi	-9.53	2.92%	2.29%	3.22%
Austria	-7.90	3.62%	3.43%	4.27%	Malaysia	-6.76	4.79%	5.33%	5.99%
Azerbaijan	-9.52	4.01%	3.15%	3.95%	Malta	-9.16	10.85%	8.80%	9.25%
Bahrain	-10.04	4.41%	3.27%	3.92%	Mauritius	-6.73	6.41%	7.16%	8.92%
Bangladesh	-6.64	2.00%	2.26%	2.89%	Mexico	-7.91	2.01%	1.90%	2.44%
Belarus	-8.20	2.78%	2.54%	3.14%	Moldova	-8.72	5.91%	5.06%	5.86%
Belgium	-9.50	5.25%	4.12%	5.39%	Mongolia	-10.17	4.98%	3.65%	4.44%
Belize	-5.69	4.12%	5.46%	6.20%	Morocco	-5.62	2.61%	3.49%	4.02%
Benin	-9.97	7.77%	5.79%	9.07%	Mozambique	-8.53	4.88%	4.28%	6.01%
Bolivia	-8.63	2.38%	2.07%	2.69%	Namibia	-8.91	3.78%	3.18%	3.82%
Botswana	-9.91	3.62%	2.73%	3.59%	Nepal	-7.84	2.51%	2.40%	3.01%
Brazil	-4.51	0.85%	1.41%	1.63%	Netherlands	-7.08	2.47%	2.62%	3.37%
Bulgaria	-7.87	3.86%	3.67%	4.43%	New Zealand	-5.88	1.60%	2.05%	2.45%
Burkina Faso	-7.26	2.54%	2.63%	3.40%	Nicaragua	-8.31	4.52%	4.07%	5.14%
Cambodia	-8.29	5.54%	5.00%	5.73%	Nigeria	-8.26	3.54%	3.21%	3.87%
Cameroon	-5.34	1.58%	2.22%	3.02%	Norway	-6.48	2.48%	2.87%	3.52%
Canada	-7.01	2.06%	2.20%	2.68%	Oman	-9.57	4.82%	3.76%	4.60%
Chile	-7.29	2.12%	2.18%	2.75%	Pakistan	-7.82	1.93%	1.85%	2.46%
China	-4.59	1.42%	2.33%	2.73%	Panama	-9.68	3.99%	3.08%	3.67%
Colombia	-7.06	1.34%	1.42%	1.91%	Paraguay	-9.10	3.27%	2.68%	3.37%
Costa Rica	-8.24	3.77%	3.43%	4.50%	Peru	-4.42	1.24%	2.12%	2.42%
Cote d'Ivoire	-6.72	2.54%	2.83%	4.16%	Philippines	-7.28	3.37%	3.47%	4.40%
Croatia	-7.00	2.92%	3.14%	3.78%	Poland	-6.60	2.45%	2.79%	3.34%
Cyprus	-7.88	4.65%	4.42%	5.33%	Portugal	-6.28	2.30%	2.75%	3.32%
Czech Republic	-6.60	3.60%	4.10%	4.83%	Qatar	-7.22	3.03%	3.15%	3.66%
Denmark	-6.77	3.64%	4.04%	4.83%	Romania	-6.99	2.48%	2.66%	3.23%
Ecuador	-7.37	2.40%	2.45%	3.15%	Russia	-6.47	1.71%	1.98%	2.41%
Egypt	-6.59	2.92%	3.32%	4.22%	Rwanda	-7.75	2.68%	2.60%	3.06%
El Salvador	-6.54	3.17%	3.64%	4.22%	Saudi Arabia	-6.36	4.89%	5.79%	6.64%
Estonia	-8.85	4.72%	3.99%	4.85%	Senegal	-7.66	4.49%	4.39%	6.37%
Ethiopia	-8.27	2.63%	2.38%	2.91%	Singapore	-6.91	6.02%	6.55%	7.65%
Finland	-5.66	2.52%	3.36%	3.85%	Slovakia	-7.78	3.66%	3.52%	4.21%
France	-5.91	1.65%	2.09%	2.51%	Slovenia	-7.64	4.17%	4.09%	4.95%
Georgia	-9.20	3.72%	3.02%	3.75%	South Africa	-5.52	1.60%	2.18%	2.65%
Germany	-6.10	2.49%	3.07%	3.73%	South Korea	-4.93	2.16%	3.30%	3.81%
Ghana	-7.72	3.30%	3.20%	4.11%	Spain	-5.83	1.95%	2.51%	2.97%
Greece	-6.98	2.77%	2.97%	3.56%	Sri Lanka	-7.07	3.09%	3.29%	4.47%
Guatemala	-8.15	3.17%	2.91%	3.78%	Sweden	-6.17	2.62%	3.20%	3.87%
Guinea	-8.34	4.43%	3.97%	5.16%	Switzerland	-9.73	3.97%	3.04%	3.95%
Honduras	-8.46	4.71%	4.16%	5.03%	Taiwan	-6.51	3.93%	4.54%	5.30%
Hong Kong	-8.53	4.18%	3.66%	4.62%	Tanzania	-9.66	3.37%	2.60%	3.29%
Hungary	-6.70	4.29%	4.82%	5.54%	Thailand	-6.69	3.93%	4.42%	5.16%
Iceland	-7.37	3.27%	3.33%	3.97%	Togo	-9.48	11.24%	8.79%	10.66%
India	-5.09	1.34%	1.98%	2.32%	Tunisia	-7.90	4.65%	4.41%	5.30%
Indonesia	-5.91	1.96%	2.49%	3.30%	Turkey	-6.53	1.89%	2.17%	2.63%
Iran	-6.94	2.70%	2.92%	3.54%	UAE	-9.28	5.93%	4.77%	5.95%
Ireland	-6.70	5.44%	6.11%	6.63%	Uganda	-8.29	2.45%	2.22%	2.82%
Israel	-7.21	2.77%	2.89%	3.57%	Ukraine	-7.95	2.77%	2.61%	3.16%
Italy	-4.68	1.57%	2.53%	2.87%	United Kingdom	-6.33	1.92%	2.27%	2.76%
Japan	-4.70	0.92%	1.48%	1.76%	United States	-5.20	0.99%	1.44%	1.71%
Kazakhstan	-7.29	2.52%	2.59%	3.14%	Uruguay	-7.81	2.44%	2.34%	2.94%
Kenya	-7.28	2.28%	2.35%	3.31%	Venezuela	-6.73	1.60%	1.78%	2.43%
Kuwait	-7.85	4.04%	3.85%	4.78%	Vietnam	-7.98	7.30%	6.85%	7.97%
Kyrgyzstan	-9.40	4.37%	3.47%	4.10%	Zambia	-9.44	1.95%	1.55%	2.00%
Laos	-9.44	2.68%	2.12%	2.68%	Zimbabwe	-7.81	6.60%	6.33%	7.79%

many more parameters are needed than in the previous calculation, as equilibrium wages must be calculated. In Appendix I, we provide details of how this computational analysis done. In particular, we need the vector of preference weights $\alpha(i)$, sector-level trade costs $\tau(i)$, and population size L_0. In addition, we need the absolute advantage parameter in the Fréchet distribution T_n. We select these parameters for each country to exactly match sector-level imports, sector-level production, population as a fraction of world population, and GDP as a fraction of world GDP. Given these parameters, we then solve for the compensating variation that equates utility in the observed trade equilibrium and autarky equilibrium. The resulting welfare gains appear in the True Gains column of Table 2.

For comparison, the ACR Gains columns list the gains from trade implied by the ACR formula under two different implementations of the ACR formula. In "ACR Gains (constant)" we apply the ACR formula with the same elasticity, which is the average trade elasticity of -7.5, in all countries. Then differences in gains from trade are due only to differences in import penetration across countries. In "ACR Gains (marginal)" we use each country's import penetration ratio, and their value of the trade elasticity measured using equation (8). These two exercises have different interpretations. Comparing the first measure to the true gains is informative about how important, overall, is it to take into account both that trade elasticities vary across countries, and that they vary over levels of trade costs. Comparing the second measure to the true gains isolates the question of the importance of variation in trade elasticities over trade costs.

Since trade elasticities are closer to zero near autarky than at observed levels of trade, Propositions 3 and 4 tell us that the True Gains entries should be higher than the ACR Gains (marginal) entries for each country. In thirteen countries the ACR Gains (constant) entry is higher than the True Gains, which occurs when a country's variable elasticity is much lower (further from zero) than -7.5. These countries are small and highly open to trade. In total they account for 1.4% of world population and have an average import penetration of 33.3%. The population-weighted average difference between the actual gains from trade are 28% greater than ACR Gains (constant), and 24% greater than the ACR Gains (marginal). As before, those countries with the lowest import penetration have the largest disagreement between the ACR prediction and the actual gains. This is because of the non-linear effect that trade has on real income. The majority of gains are realized near autarky, so countries with higher import penetration realize a diminishing welfare effect.

6 Conclusion

In this paper we provide a method for measuring gains from trade in models that exhibit a variable trade elasticity. We find an important role for reallocation of expenditure across industries in response to changes in trade costs that is absent in models that assume constant expenditure shares across industries. This effect makes trade more inelastic as the economy approaches autarky, and therefore means that marginal increases in import penetration near autarky have a larger welfare effect than those near observed levels of trade. This is untrue in models with constant trade elasticities, where the marginal impact of import penetration on welfare is constant.

The methodology developed in this paper is intended to be general and applicable to a variety of models. These tools allow researchers to measure trade elasticities even in more complex, non-homothetic models, and to measure gains from trade that are easily comparable to more standard models.

In future work, we will focus on cases with heterogeneous households. Because our tools allow for non-homotheticities, within-population differences in income may matter for both patterns of consumption and responses to changes in trade costs. The unequal effects of trade across the set of households is an important area of ongoing research (for instance, see Fajgelbaum and Khandelwal, 2016), and it may be possible to extent our results to contribute to this area.

References

Adao, Rodrigo, Arnaud Costinot, and Dave Donaldson. 2017. "Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade." *American Economic Review*, 107(3): 633–689.

Alessandria, George, and Horag Choi. 2014*a*. "Do Falling Iceberg Costs Explain U.S. Export Growth?" *Journal of International Economics*, 94(2): 311–325.

Alessandria, George, and Horag Choi. 2014*b*. "Establishment Heterogeneity, Exporter Dynamics, and the Effects of Trade Liberalization." *Journal of International Economics*, 94(2): 207–223.

Alessandria, George, Horag Choi, and Kim Ruhl. 2014. "Trade Adjustment Dynamics and the Welfare Gains from Trade." NBER Working Paper 20663.

Allen, Treb, Costas Arkolakis, and Yuta Takahashi. 2014. "Universal Gravity." NBER Working Paper No. 20787.

Anderson, James E., and Eric Van Wincoop. 2004. "Trade Costs." *Journal of Economic Literature*, 42(3): 691–751.

Ardelean, Adina, and Volodymyr Lugoskyy. 2010. "Domestic Productivity and Variety Gains from Trade." *Journal of International Economics*, 80(2): 280–291.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. "New Trade Models, Same Old Gains?" *American Economic Review*, 102(1): 94–130.

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson, and Andrés Rodríguez-Clare. forthcoming. "The Elusive Pro-Competitive Effects of Trade." *Review of Economic Studies*.

Baier, Scott L., and Jeffrey H. Bergstrand. 2009. "Estimating the Effects of Free Trade Agreements on International Trade Flows Using Matching Econometrics." *Journal of International Economics*, 77(1): 63–76.

Baier, Scott L., Jeffrey H. Bergstrand, and Michael Feng. 2014. "Economic Integration Agreements and the Margins of International Trade." *Journal of International Economics*, 93(2): 339–350.

Blonigen, Bruce A., and Anson Soderbery. 2010. "Measuring the Benefits of Foreign Product Variety with an Accurate Variety Set." *Journal of International Economics*, 82(2): 168–180.

Broda, Christian, and David E. Weinstein. 2006. "Globalization and Gains from Variety." *Quarterly Journal of Economics*, 121(2): 541–585.

Brooks, Wyatt J., and Pau S. Pujolas. 2017. "Capital Accumulation and the Welfare Gains from Trade." *Economic Theory.* Forthcoming.

Caron, Justin, Thibault Fally, and James R. Markusen. 2014. "International Trade Puzzles: A Solution Linking Production and Preferences." *Quarterly Journal of Economics*, 129(3): 1501–1552.

Chaney, Thomas. 2008. "Distorted Gravity: the Intensive and Extensive Margins of International Trade." *American Economic Review*, 98(4): 1707–1721.

Cooke, Richard G. 2014. *Infinite Matrices and Sequence Spaces.* Dover Publishers, Mineola, NY.

Costinot, Arnaud, and Andrés Rodríguez-Clare. 2014. "Trade Theory with Numbers: Quantifying the Consequences of Globalization." *Handbook of International Economics*, 4: 197–261. Gita Gopinath, Elhanan Helpman and Kenneth Rogoff, eds.

Dhingra, Swati, and John Morrow. forthcoming. "Monopolistic Competition and Optimum Product Diversity Under Firm Heterogeneity." *Journal of Political Economy.*

Eaton, Jonathon, and Samuel Kortum. 2002. "Technology, Geography and Trade." *Econometrica*, 70(5): 1741–1779.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu. 2015. "Competition, Markups and the Gains from International Trade." *American Economic Review*, 105(10): 3183–3221.

Fajgelbaum, Pablo D., and Amit K. Khandelwal. 2016. "Measuring the Unequal Gains from Trade." *Quarterly Journal of Economics*, 131(3): 1113–1180.

Feenstra, Robert C. 2018. "Restoring the Product Variety and Pro-competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity." *Journal of International Economics*, 110(1): 16–27.

Feenstra, Robert C., and David E. Weinstein. 2017. "Globalization, Markups and U.S. Welfare." *Journal of Political Economy*, 125(4): 1040–1074.

Fieler, Ana Cecilia. 2011. "Nonhomotheticity and Bilateral Trade: Evidence and a Quantitative Explanation." *Econometrica*, 79(4): 1069–1101.

Holmes, Thomas J., Wen-Tai Hsu, and Sanghoon Lee. 2014. "Allocative Efficiency, Mark-ups and the Welfare Gains from Trade." *Journal of International Economics*, 94(2): 195–206.

Krugman, Paul. 1980. "Scale Economies, Product Differentiation and the Pattern of Trade." *American Economic Review*, 70(5): 950–959.

Markusen, James R. 1986. "Explaining the Volume of Trade: An Ecletic Approach." *American Economic Review*, 76(5): 1002–1011.

Melitz, Marc J. 2003. "The Impact of Trade on Aggregate Industry Productivity and Intra-Industry Reallocation." *Econometrica*, 71(6): 1695–1725.

Melitz, Marc J., and Stephen J. Redding. 2014. "Missing Gains from Trade?" *American Economic Review*, 104(5): 317–321.

Melitz, Marc J., and Stephen J. Redding. 2015. "New Trade Models, New Welfare Implications." *American Economic Review*, 105(3): 1105–1146.

Mrázová, Monika, and J. Peter Neary. 2017. "Not So Demanding: Demand Structure and Firm Behavior." *American Economic Review*, 107(12): 3835–3874.

Narayanan, Badri G., Angel Aguiar, and Robert McDougall, ed. 2012. *Global Trade, Assistance, and Production: The GTAP 8 Data Base.* Center for Global Trade Analysis, Purdue University. Available online at: https://www.gtap.agecon.purdue.edu/databases/v7/v7_doco.asp.

Ossa, Ralph. 2015. "Why Trade Matters After All." *Journal of International Economics*, 97(2): 266–277.

Ruhl, Kim J. 2008. "The International Elasticity Puzzle." working paper.

Simonovska, Ina. 2015. "Income Differences and Prices of Tradables: Insights from an Online Retailer." *Review of Economic Studies*, 82(4): 1612–1656.

Simonovska, Ina, and Michael E. Waugh. 2014. "The Elasticity of Trade: Estimates and Evidence." *Journal of International Economics*, 92(1): 34–50.

Zhelobodko, Evgeny, Sergey Kokovin, Mathieu Parenti, and Thisse-Jacques-François. 2012. "Monopolistic Competition: Beyond the Constant Elasticity of Substitution." *Econometrica*, 80(6): 2765–2784.

Appendix

A Multiple Trading Partners

We now analyze the effects on country 0 of changing trade costs with its N other trading partners, which we index $n = 1, ..., N$. We assume that a parameter τ governs trade costs between country 0 and all its trading partners.[8] With more trading partners, we need to change the definitions of the elasticities. First, we define the trade elasticity as:

$$\varepsilon_T = \frac{\frac{\partial \log\left(\frac{1-\lambda_{00}}{\lambda_{00}}\right)}{\partial \log(\tau)}}{\sum_{n=1}^{N} \frac{\lambda_{0n}}{1-\lambda_{00}} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)}$$

The demand elasticities $\kappa(i)$ have the same definition as before, but the definition of the production elasticity is now:

$$\rho(i) = \frac{\frac{\partial \log\left(\frac{1-\lambda_{00}(i)}{\lambda_{00}(i)}\right)}{\partial \log(\tau)}}{\sum_{n=1}^{N} \frac{\lambda_{0n}(i)}{1-\lambda_{00}(i)} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)}$$

Notice that if all N trading partners are identical, then this trade elasticity is the same as the case of a single trading partner.

Proposition 5 *Whenever $\lambda_{00} \in (0, 1)$,*

$$\begin{aligned}\varepsilon_T = &\int_{\Omega} [\rho(i) - (1 + \kappa(i))] \frac{\sum_{n=1}^{N} \lambda_{0n}(i) \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)}{\sum_{n=1}^{N} \lambda_{0n} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)} \frac{X_{00}(i)}{X_{00}} di \\ &+ \int_{\Omega} (1 + \kappa(i)) \frac{\sum_{n=1}^{N} \lambda_{0n}(i) \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)}{\sum_{n=1}^{N} \lambda_{0n} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)} X_0(i) di \frac{\int_{\Omega} \kappa(i) \frac{X_{00}(i)}{X_{00}} di}{\int_{\Omega} \kappa(i) X_0(i) di}\end{aligned} \tag{23}$$

Proof *First, note that the analogue of Lemma 7 now is:*

$$\frac{\partial \log(p(i))}{\partial \log(\tau)} = \sum_{n=1}^{N} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right) \lambda_{0n}(i)$$

In Appendix D we prove Proposition 1 of the main text. Notice that nothing in the proof presented in Appendix D for Proposition 1 is changed up to this equation:

$$\begin{aligned}\frac{\partial \log(\lambda_{00})}{\partial \log(\tau)} = &\int_{\Omega} \left[\frac{\partial \log(\lambda_{00}(i))}{\partial \log(\tau)} + (1 + \kappa(i)) \frac{\partial \log(p_0(i))}{\partial \log(\tau)}\right] \frac{X_{00}(i)}{X_{00}} di \\ &- \int_{\Omega} \kappa(i) \frac{X_{00}(i)}{X_{00}} di \frac{\int_{\Omega} \frac{\partial \log(p_0(i))}{\partial \log(\tau)} (1 + \kappa(i)) X_0(i) di}{\int_{\Omega} \kappa(i) X_0(i) di}\end{aligned} \tag{24}$$

Then the result follows immediately by substituting in the above equation for price changes and the definition of ε_T.

[8]That is, $\tau_{0j} = \tilde{\tau}_{0j}\tau$ and we will be considering changes in the common component τ.

B Non-Separable Preferences

For simplicity in the body of the paper we assumed that the utility function was additively seperable. In this section we dispense with that assumption and show that our results are unchanged. That is, the household's problem is now:

$$\max U\left(\{\{c_n(i)\}_{i\in\Omega_n}\}_{n=0}^{N}\right)$$
$$s.t. \quad : \quad \sum_{n=0}^{N} \tau_{0n} p_n(i) c_n(i) \leq I_0$$

This specification allows for complementarity or substitutability of goods both within and between countries.

Let H be the Hessian matrix of U. Because U is strictly concave and twice continuously differentiable, H is negative definite and invertible.[9]

Row i of H contains all the second order partial derivatives of good i with all other goods.

The analogous demand elasticity to what we had before is:

$$\beta(i,j) = H^{-1}_{(i,j)} \frac{\frac{\partial U}{\partial c(i)}}{c(j)}$$

where $H^{-1}_{(i,j)}$ is the (i,j) entry in the inverse of H. Notice that if U was additively separable as before, then H^{-1} is a diagonal matrix where the (i,i) entry is the reciprocal of the second derivative with respect to good i of the utility function. This implies, $\forall i, \beta(i,i) = \kappa(i)$ and $\forall j \neq i, \beta(i,j) = 0$.

The general case of Proposition 1 is as follows:

Proposition 6 *Whenever* $\lambda_{00} \in (0,1)$,

$$\varepsilon_T = \int_\Omega (\rho(i)-1) \frac{\frac{X_{00}(i)}{X_{00}} \frac{X_{01}(i)}{X_{01}}}{\frac{X_0(i)}{I_0}} di - \int_\Omega \int_\Omega \beta(i,j) \frac{\lambda_{01}(i)}{\lambda_{01}} \frac{X_{00}(j)}{X_{00}} didj$$
$$+ \int_\Omega \int_\Omega \beta(i,j) \frac{X_{00}(j)}{X_{00}} didj \frac{1 + \int_\Omega \int_\Omega \beta(i,j) \frac{X_0(j)}{X_0(i)} \frac{X_{01}(i)}{X_{01}} didj}{\int_\Omega \int_\Omega \beta(i,j) \frac{X_0(j)}{I_0} didj}$$

Proof *By the definition of* λ_{00}*:*

$$\lambda_{00} I_0 = \int_\Omega \lambda_{00}(i) p(i) c(i) di$$

Differentiating with respect to $\log(\tau)$ *and using the definition of* ε_T *implies:*

$$\begin{aligned} \varepsilon_T &= -\frac{\int_\Omega \left(\frac{\partial \log(\lambda_{00}(i))}{\partial \log(\tau)} + \frac{\partial \log(p(i))}{\partial \log(\tau)} + \frac{\partial \log(c(i))}{\partial \log(\tau)}\right) X_{00}(i) di}{\lambda_{01} X_{00} \left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau)}\right)} \\ &= \int_\Omega (\rho(i)-1) \frac{\frac{X_{00}(i)}{X_{00}} \frac{X_{01}(i)}{X_{01}}}{\frac{X_0(i)}{I_0}} di - \int_\Omega \frac{\frac{\partial \log(c(i))}{\partial \log(\tau)}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau)}} \frac{X_{00}(i)}{\lambda_{01} X_{00}} di \end{aligned}$$

[9] If H is an infinite matrix, additional regularly assumptions may be necessary. In that case, by inverse we mean the left-hand reciprocal of H, as described in Cooke (2014).

Note that first order conditions for the household are:

$$\frac{\partial U}{\partial c(i)} = \mu_0 p(i)$$

The left hand side may depend on goods other than good i. Therefore, when we differentiate these first order conditions with respect to $\log(\tau)$ we apply the chain rule to get:

$$\int_\Omega \frac{\partial c(j)}{\partial \log(\tau)} \frac{\partial^2 U}{\partial c(i)\partial c(j)} dj = \mu_0 p(i) \left(\frac{\partial \log(\mu_0)}{\partial \log(\tau)} + \frac{\partial \log(p(i))}{\partial \log(\tau)} \right)$$

For H as defined above, this can be used to solve for changes in consumption as follows:

$$\frac{\partial \log(c(j))}{\partial \log(\tau)} = \int_\Omega \beta(i,j) \frac{\partial \log(p(i))}{\partial \log(\tau)} di + \frac{\partial \log(\mu_0)}{\partial \log(\tau)} \int_\Omega \beta(i,j) di$$

Differentiating the budget constraint of the household with respect to $\log(\tau)$ allows us to solve for changes in μ_0:

$$\frac{\partial \log(\mu_0)}{\partial \log(\tau)} = -\left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau)}\right) \frac{1 + \int_\Omega \int_\Omega \beta(i,j)\lambda_{01}(i) X_0(j) di dj}{\int_\Omega \int_\Omega \beta(i,j) X_0(j) di dj}$$

Substituting the derivative of μ_0 into the derivative of $c(j)$ and substituting that into the formula for ε_T yields the result.

Note that Propositions 2 and 4 go through in this environment with no changes.

C Demand Systems instead of Utility Functions

Now suppose that the demand side of the economy is modeled using a demand system. In this case, we do not assume the existence of a utility function. Instead we assume that expenditure on sector j in country 0 is given by:

$$X_0(j) = X_{0j}(\{p_{0k}\}) \tag{25}$$

We do assume that this demand system does satisfy household budget constraints so that:

$$w_0 L_0 + \Pi_0 = \int_\Omega X_0(j) dj \tag{26}$$

where Π_0 is aggregate profit in the domestic country.

In this case, the aggregate trade elasticity is given by:

$$\varepsilon_T = \int_\Omega \rho(i) \frac{\omega_{01}(i)\omega_{00}(i)}{\omega_0(i)} di + \int_\Omega \left([\omega_{01}(j) - \omega_{00}(j)] \int_\Omega \lambda_{01}(k) \frac{\partial \log(X_{0j})}{\partial \log(p_{0k})} dk \right) dj \tag{27}$$

Notice that two of the results discussed before are preserved in this case. First, if import penetration ratios are constant in all sectors, so that $\omega_{00}(i) = \omega_{01}(i)$ for all i, then the second term is clearly zero. Second, if expenditure shares are constant, then $\partial \log(X_{0j})/\partial \log(p_{0k}) = 0$ for all j and k. Again, this implies that the second term is zero.

D Proof of Proposition 1

In the appendix we write τ_{01} as τ to save notation. Rewriting the definition of λ_{00} yields:

$$\lambda_{00} I_0 = \int_\Omega \lambda_{00}(i) p_0(i) c_0(i) di$$

Noting that the first order condition is:

$$u'(c(i), i) = p(i)\mu_0 + \nu_0(i)$$

Here μ_0 is the Lagrange multiplier on the country 0 household budget constraint and $\nu_0(i)$ is the Lagrange multiplier on the non-negativity constraint for good i. Note that:

$$\nu_0(i) = \begin{cases} 0 & \text{if } c(i) > 0 \\ u'(0, i) - p(i)\mu_0 & \text{if } c(i) = 0 \end{cases}$$

Then:

$$\frac{\partial \nu_0(i)}{\partial \log(\tau)} = \begin{cases} 0 & \text{if } c(i) > 0 \\ -p(i)\mu_0 \left[\frac{\partial \log(p(i))}{\partial \log(\tau)} + \frac{\partial \log(\mu_0)}{\partial \log(\tau)} \right] & \text{if } c(i) = 0 \end{cases}$$

Therefore we can solve for changes in consumption as:

$$u''(c(i), i) \frac{\partial c(i)}{\partial \log(\tau)} = \begin{cases} u'(c(i), i) \left[\frac{\partial \log(p(i))}{\partial \log(\tau)} + \frac{\partial \log(\mu_0)}{\partial \log(\tau)} \right] & \text{if } c(i) > 0 \\ 0 & \text{if } c(i) = 0 \end{cases}$$

Then using the notation from Section 3, we can write this as:

$$\frac{\partial \log(c(i))}{\partial \log(\tau)} = \begin{cases} \kappa(i) \left[\frac{\partial \log(p(i))}{\partial \log(\tau)} + \frac{\partial \log(\mu_0)}{\partial \log(\tau)} \right] & \text{if } c(i) > 0 \\ 0 & \text{if } c(i) = 0 \end{cases}$$

Now we can differentate the definition of λ_{00} and get:

$$\frac{\partial \log(\lambda_{00})}{\partial \log(\tau)} = \int_\Omega \left[\frac{\partial \log(\lambda_{00}(i))}{\partial \log(\tau)} + (1 + \kappa(i)) \frac{\partial \log(p_0(i))}{\partial \log(\tau)} + \kappa(i) \frac{\partial \log(\mu_0)}{\partial \log(\tau)} \right] \frac{X_{00}(i)}{X_{00}} di$$

Then the budget constraint of the household implies:

$$\frac{\partial \log(\mu_0)}{\partial \log(\tau)} = -\frac{\int_\Omega \frac{\partial \log(p_0(i))}{\partial \log(\tau)} (1 + \kappa(i)) X_0(i) di}{\int_\Omega \kappa(i) X_0(i) di}$$

Substituting in the change in μ_0 term implies:

$$\begin{aligned} \frac{\partial \log(\lambda_{00})}{\partial \log(\tau)} = & \int_\Omega \left[\frac{\partial \log(\lambda_{00}(i))}{\partial \log(\tau)} + (1 + \kappa(i)) \frac{\partial \log(p_0(i))}{\partial \log(\tau)} \right] \frac{X_{00}(i)}{X_{00}} di \\ & - \int_\Omega \kappa(i) \frac{X_{00}(i)}{X_{00}} di \frac{\int_\Omega \frac{\partial \log(p_0(i))}{\partial \log(\tau)} (1+\kappa(i)) X_0(i) di}{\int_\Omega \kappa(i) X_0(i) di} \end{aligned}$$

Recall that the trade elasticity can be rewritten as:

$$\varepsilon_T = \frac{\frac{\partial \log\left(\frac{1-\lambda_{00}}{\lambda_{00}}\right)}{\partial \log(\tau)}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau)}} = -\frac{1}{1-\lambda_{00}} \frac{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau)}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau)}}$$

Then using the results above and Lemma 7 yield:

$$\varepsilon_T = \int_\Omega \left[\rho(i) - (1+\kappa(i))\right] \frac{\lambda_{01}(i)}{\lambda_{01}} \frac{X_{00}(i)}{X_{00}} di + \int_\Omega (1+\kappa(i)) \frac{X_{01}(i)}{\lambda_{01}} di \frac{\int_\Omega \kappa(i) \frac{X_{00}(i)}{X_{00}} di}{\int_\Omega \kappa(i) X_0(i) di}$$

This is equivalent to the result.

E Examples

In Table 3 we list a number of examples of how to compute preference elasticities, $\kappa(i)$, and production elasticities, $\rho(i)$.

Table 3: Examples of Elasticity Values in Different Environments

Preference type	u_i	κ_i
Cobb-Douglas (in logs)	$\alpha_i \log(c_i)$	-1
Constant Elasticity of Substitution	$\alpha_i c_i^{1-1/\sigma}$	$-\sigma$
Constant Relative Income Elasticity	$\alpha_i c_i^{1-1/\sigma_i}$	$-\sigma_i$
Stone-Geary	$\alpha(i) \log(c(i) - \gamma(i))$	$\frac{\gamma(i)}{c(i)} - 1$
Hyperbolic Absolute Risk Aversion	$(\alpha_i c_i + \gamma_i)^{\sigma_i}$	$\frac{1+\frac{\gamma_i}{\alpha_i c_i}}{\sigma_i - 1}$
Production type	F_i	ρ_i
Constant Elasticity of Substitution	$\left(\alpha_i y_{0i}^{\frac{\gamma_i-1}{\gamma_i}} + (1-\alpha_i) y_{1i}^{\frac{\gamma_i-1}{\gamma_i}}\right)^{\frac{\gamma_i}{\gamma_i-1}}$	$1-\gamma_i$
Variable Elasticity of Substitution	$\left(\alpha_i y_{0i}^{\frac{\gamma_i-1}{\gamma_i}} + (1-\alpha_i) y_{0i}^{\eta_i} y_{1i}^{\frac{\gamma_i-1}{\gamma_i}-\eta_i}\right)^{\frac{\gamma_i}{\gamma_i-1}}$	$1 - \frac{1-\left(\frac{\eta_i\gamma_i}{\gamma_i-1}\right)^2 - (1-\frac{\eta_i\gamma_i}{\gamma_i-1})\lambda_{00}(i)}{\eta_i + \gamma_i \frac{1-\eta_i^2}{(\gamma_i-1)^2} + \frac{\eta_i(\gamma_i-1)-\gamma_i}{(\gamma_i-1)^2}\lambda_{00}(i)}$
Eaton-Kortum per sector	Fréchet distribution param. $\theta(i)$	$-\theta(i)$

F Proof of Proposition 2

First note that $\lambda_{00} + \lambda_{01} = 1$, so that

$$\frac{\partial \lambda_{00}}{\partial \log(\tau_{01})} = -\frac{\partial \lambda_{01}}{\partial \log(\tau_{01})} \implies \lambda_{00} \frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})} = -\lambda_{01} \frac{\partial \log(\lambda_{01})}{\partial \log(\tau_{01})}$$

Then we can rewrite the definition of the trade elasticity:

$$\varepsilon_T = \frac{\frac{\partial \log(\lambda_{01}/\lambda_{00})}{\partial \log(\tau_{01})}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}} = \frac{\frac{\partial \log(\lambda_{01})}{\partial \log(\tau_{01})} - \frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}} = -\frac{1}{1-\lambda_{00}} \frac{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}}$$

Using the envelope theorem on the dual of the consumer's problem written above yields:

$$\frac{\partial \log(I)}{\partial \log(\tau_{01})} = \int_{\Omega} \frac{\partial \log(p(i))}{\partial \log(\tau_{01})} \frac{p(i)c(i)}{I_0} di$$

In the following Lemma we characterize how changes in prices change with trade costs, which we will require to continue with the proof.

Lemma 7

$$\frac{\partial \log(p(i))}{\partial \log(\tau_{01})} = \left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}\right) \lambda_{01}(i)$$

The proof of the lemma follows immediately from the envelope theorem applied to the final good producing firm's problem, and the assumption that intermediate good prices are linear in wages. Using the lemma implies:

$$\frac{\partial \log(I)}{\partial \log(\tau_{01})} = \left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}\right) \int_{\Omega} \frac{X_{01}(i)}{I_0} di = \left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}\right)(1 - \lambda_{00})$$

$$\frac{1}{\varepsilon_W} = -\frac{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}{\frac{\partial \log(I)}{\partial \log(\tau_{01})}} = -\frac{1}{1-\lambda_{00}} \frac{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}} = \varepsilon_T$$

This completes the proof.

G Proof of Proposition 3

Note that $\forall \tau, \lambda_{00} \in (0,1) \implies \log(\lambda_{00}) < 0$, and clearly $\varepsilon_T(\tau)^2 > 0$. Therefore, for all τ,

$$sign\left(-\frac{\log(\lambda_{00})}{\varepsilon_T(\tau)^2} \frac{\partial \varepsilon_T}{\partial \log(\tau)}\right) = sign\left(\frac{\partial \varepsilon_T}{\partial \log(\tau)}\right)$$

Suppose that ε_T is increasing in τ. Then the term within the integral is positive for all τ, hence:

$$0 < \int_{\tau_{01}}^{\tau_{AUT}} -\frac{\log(\lambda_{00})}{\varepsilon_T(\tau)^2} \frac{\partial \varepsilon_T}{\partial \log(\tau)} d\log(\tau)$$

Together with equation (13), this implies the result. The same argument goes through if ε_T is decreasing in τ_{01}.

H Proof of Proposition 4

Suppose a set of sectors S all have the minimum value of θ such that $\forall i \in S : \theta(i) = \theta^{MIN}$. The sector-level trade elasticity is $-\theta(i)$, so as trade costs get large, low $\theta(i)$ sectors become a larger share of aggregate imports. In the limit, imports from the sectors in S approach one hundred percent of total imports. That is,

$$\lim_{\tau_{01} \to \infty} \sum_{i \in S} \frac{X_{01}(i)}{X_{01}} = 1 \quad \text{and} \quad \lim_{\tau_{01} \to \infty} \sum_{i \notin S} \frac{X_{01}(i)}{X_{01}} = 0$$

Likewise,

$$\forall i, \lim_{\tau_{01}\to\infty} X_{01}(i) = 0 \implies \forall i \lim_{\tau_{01}\to\infty} \frac{X_{00}(i)}{X_0(i)} = \lim_{\tau_{01}\to\infty} \frac{X_{00}}{I_0} = 1$$

Therefore,

$$i \notin S \implies \lim_{\tau_{01}\to\infty} \frac{\frac{X_{01}(i)}{X_{01}} \frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}} = 0 \quad \text{and} \quad \forall i, \lim_{\tau_{01}\to\infty} \frac{\sum_i \sigma(i) \frac{X_{00}(i)}{X_{00}}}{\sum_i \sigma(i) \frac{X_0(i)}{I_0}} = 1$$

Then:

$$\lim_{\tau_{01}\to\infty} \sum_{i\in S} \theta(i) \frac{\frac{X_{01}(i)}{X_{01}} \frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}} = \lim_{\tau_{01}\to\infty} \theta^{MIN} \sum_{i\in S} \frac{\frac{X_{01}(i)}{X_{01}} \frac{X_{00}(i)}{X_0(i)}}{\frac{X_{00}}{I_0}} = \theta^{MIN}$$

and

$$\lim_{\tau_{01}\to\infty} \sum_{i\in S} (\sigma(i) - 1) \frac{X_{01}(i)}{X_{01}} \left[\frac{\frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}} - \frac{\sum_i \sigma(i) \frac{X_{00}(i)}{X_{00}}}{\sum_i \sigma(i) \frac{X_0(i)}{I_0}} \right] = 0$$

Combining these two equations into equation (8) implies the result.

I Computational Appendix

I.1 Data

This exercise makes use of two data sources. The first is the GTAP 8 database (Narayanan, Aguiar and McDougall, 2012). We consider the 50 sectors that Caron, Fally and Markusen (2014) produce estimates for. We compute the sectoral share of total consumption in each sector for each country. We also compute each country's share of world GDP and world population. We then compute the sectoral composition of imports and exports. Here we have to account for the fact that the model is static, and that we assume that trade is balanced. To implement this, we use total trade as a fraction of gross output, then assume that total trade is half imports and half exports. Then the volume of exports in each sector is equal to the imputed total exports multiplied by the sectoral share of exports from the data. Imports are computed in the same way. Therefore, our exercise uses the exact sectoral composition of imports and exports from the data, but total imports and total exports differ from the data in model by the magnitude of the trade imbalance each country is running.

The second data input is the set of elasticity estimates derived from Caron, Fally and Markusen (2014). This paper estimates separate production and consumption elasticities for each of 50 sectors in the GTAP database.

Another challenge in solving this model is that the expenditures and trade flows in all sectors are assumed in to be positive. In particular, zero expenditure in sector j would imply that the preference parameter $\alpha_0(j) = 0$, but the trade cost τ_{01} would not be identified. As trade goes to zero with positive expenditure, that implies τ_{01} tends toward infinity. Therefore, identification of all parameters is not possible in either of those cases. We get around this issue by including very small values of trade flows and expenditure in all sectors that exhibit zeros.[10]

[10] For every sector with zero expenditure, we set the expenditure value equal to 1/1,000 of the value in the sector with the lowest positive expenditure. We make the same correction for imports and exports. We also vary this exercise by considering 1/100 and 1/10,000 and find no noticeable difference in the results.

I.2 Parameterizing the Model

The model uses a familiar Eaton and Kortum (2002) production structure such that productivities in sector j and country n are drawn from a Frechet distribution with parameters T_n and $\theta(j)$. Varieties within sectors are aggregated with an elasticity of substitution equal to ξ. The parameter ξ does not enter the trade elasticity directly, but must satisfy $\xi - 1 < \theta(j)$ in order for an equilibrium to exist. Because $\theta(j) > 1$, we choose to set $\forall j, \xi = 2$ to guarantee this inequality is satisfied.

Following the model as written in the main text, there are four sets of unknown parameters: T_n, the country-level absolute advantage term entering the Frechet distribution, L_n, the labor endowment in each country, $\alpha_n(i)$, the preference parameter for sector i in country n, and $\tau_{nm}(i)$, the trade cost between countries n and m for good i. To simplify this analysis, we consider the two-country case where for each country n we analyze trade between country n and the rest of the world. We assume that trade costs are symmetric so that $\forall i, \tau_{nm}(i) = \tau_{mn}(i)$, and that preference parameters are equal across countries, so that $\forall i, \alpha_n(i) = \alpha_m(i)$. However, the absolute advantage terms T_n and the labor endowments L_n do vary across countries.

In what follows, we refer to the country being analyzed as country 0 and the rest of the world as country 1. The labor endowments L_n are set to match the relative population of each country to the rest of the world, with the normalization $L_0 = 1$. We also normalize the absolute advantage term in the rest of the world $T_1 = 1$.

Next we choose the trade costs $\tau_{01}(i)$, the preference parameters $\alpha_n(i)$, and the absolute advantage term in country 0, T_0, to match the following moments: the volume of imports in each sector in country 0, the share of final uses in each sector in country 0, and the relative aggregate output of country 0 compared to the rest of the world. Because the sectoral shares must add to one, and the preference parameters $\alpha_n(i)$ are only identified up to a multiplicative constant, we must normalize the preference parameter of one sector, so we set $\alpha_n(50) = 1$. Now we effectively have 100 moments to match using 100 parameters.

We then make use of the following equilibrium conditions to match the model parameters to the target moments. Using the notation in the text, we refer to $\omega_0(j)$ as the expenditure share by country 0 in sector j, and $\omega_{0n}(j)$ as the share of sector j expenditure by country 0 on goods coming from country n. First, because this environment is competitive there are no profits. Therefore, aggregate output in country n, Y_n, is given by:

$$Y_n = w_n L_n \tag{28}$$

Therefore, with w_0 as the numeraire and the normalization $L_0 = 1$, we can write:

$$\frac{w_1}{w_0} = \frac{Y_1 L_0}{Y_0 L_1} \implies w_1 = \frac{Y_1}{Y_0 L_1} \tag{29}$$

Next, we use the well-known equation from Eaton and Kortum (2002):

$$\omega_{kn}(j) = \frac{T_n(\tau_{kn}(j) w_n)^{-\theta(j)}}{\sum_{m=0}^{1} T_m(\tau_{km}(j) w_m)^{-\theta(j)}} \tag{30}$$

With the normalization $T_1 = 1$, this can then be rewritten as:

$$\tau_{01}(j) = \left(\frac{\omega_{00}(j)}{\omega_{01}(j)}\right)^{1/\theta(j)} \frac{T_0^{-1/\theta(j)}}{w_1} \tag{31}$$

Moreover, from the first order conditions of the consumer's problem in country n, we know that:

$$\mu_n p_n(j) = \alpha_n(j) \frac{\sigma(j)}{\sigma(j)-1} c_n(j)^{-1/\sigma(j)} \tag{32}$$

This can be expressed in terms of expenditures such that:

$$p_n(j)c_n(j) = p_n(j)^{1-\sigma(j)} \mu_n^{-\sigma(j)} \left(\frac{\sigma(j)-1}{\sigma(j)\alpha_n(j)}\right)^{-\sigma(j)} \tag{33}$$

Noting that $\alpha_n(50) = 1$, then $\alpha(j)$ can be solved for as:

$$\alpha_n(j) = \frac{(p_n(j)c_n(j))^{1/\sigma(j)}}{(p_n(50)c_n(50))^{1/\sigma(50)}} \frac{p_n(j)^{1-1/\sigma(j)}}{p_n(50)^{1-1/\sigma(50)}} \frac{\sigma(50)}{\sigma(j)} \frac{\sigma(j)-1}{\sigma(50)-1} \tag{34}$$

Also by the familiar arguments in Eaton and Kortum (2002), the prices in each country $p_0(j)$ and $p_1(j)$ are given by:

$$p_0(j) = \Gamma\left(\frac{\theta(j)-1}{\theta(j)}\right) \left(T_0 + (w_1 \tau_{01}(j))^{-\theta(j)}\right)^{-1/\theta(j)} \tag{35}$$

and

$$p_1(j) = \Gamma\left(\frac{\theta(j)-1}{\theta(j)}\right) \left(T_0 \tau_{01}(j)^{-\theta(j)} + w_1^{-\theta(j)}\right)^{-1/\theta(j)} \tag{36}$$

Then the budget constraint in country 1 can be written as:

$$w_1 L_1 = \sum_{j=1}^{50} p_1(j)c_1(j) = \sum_{j=1}^{50} p_1(j)^{1-\sigma(j)} \mu_1^{-\sigma(j)} \left(\frac{\sigma(j)-1}{\sigma(j)\alpha_1(j)}\right)^{-\sigma(j)} \tag{37}$$

Finally, trade is balanced in the sense that:

$$\sum_{j=1}^{50} \omega_{01}(j) p_0(j) c_0(j) = \sum_{j=1}^{50} \omega_{10}(j) p_1(j) c_1(j) \tag{38}$$

I.3 Algorithm

The algorithm to solve and parameterize the model is as follows.

First, guess values of T_0 and μ_1. The wage in country 1, w_1, is given by equation (29) as only a function of data on relative income and population.

Second, we solve for all unknown trade costs and preference weights, given those guesses, as follows. We observe the shares $\omega_{01}(j)$ and $\omega_{00}(j)$ for each sector j from the data, so, for our guess of T_0, we can then use equation (31) to solve for the set of trade costs, $\tau_{01}(j)$. Next, prices $p_n(j)$ can be solved for using equations (35) and (36). Then expenditures $p_0(j)c_0(j)$ are observed as aggregate income Y_0 multiplied by the expenditure shares from the data. Plugging those expenditures, prices and known parameters $\sigma(j)$ into equation (34) gives values for $\alpha_0(j)$.

Third, we can solve for the Lagrange multiplier μ_0 by rearranging equation (32):

$$\mu_0 = \frac{\sigma(50)}{\sigma(50)-1}\frac{p_0(50)^{1/\sigma(50)-1}}{(p_0(50)c_0(50))^{1/\sigma(50)}} \tag{39}$$

Finally, we check that the values of T_0 and μ_1 that we guessed in the first step satisfy the budget constraint in country 1 given by equation (37), and trade balance given by equation (38). If so, then the values of $\alpha_n(j)$ and $\tau_{01}(j)$ solved for in the second step, along with the guess T_0, constitute the model's unknown parameters. If not, the guesses of μ_1 and T_0 are updated and these steps are repeated.[11]

[11] Of course, here we have written pseudo-code to describe the logic of how the model is solved and the parameters identified. In practice, this is easily implemented using Newton's Method or any common nonlinear equation solver.