The Neoclassical Growth Model and the Labor Share Decline*

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Abstract

The neoclassical growth model (NGM), which is widely used to study macroeconomic phenomena, has a constant labor share built in. However, several recent studies show that the labor share has been declining since at least the 1980s. This calls into question whether the NGM should still be used as a primary ingredient of macroeconomic models. We answer the question comparing the accuracy of the NGM's predictions on macroeconomic variables with versions of the model where the parameter governing the labor share declines. Employing the Akaike and Bayesian Information Criteria, we find that the NGM with a constant parameter is preferred. A battery of robustness checks does not alter our conclusion.

Keywords: Neoclassical Growth Model, Labor Share, AIC, BIC.
JEL Codes:

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1 Introduction

The neoclassical growth model (NGM) is the workhorse of modern macroeconomics.[1] Its well-known ingredients include a rational, forward-looking, representative household that decides how much to work, invest and consume; a representative firm that produces a final good operating a Cobb-Douglas production function with capital and labor; and perfectly competitive markets. Combining these ingredients, the NGM can accurately reproduce sequences of macroeconomic aggregates that are consistent with Kaldor's *stylized facts* (Kaldor, 1957), explaining the model's success and broad appeal. In particular, the NGM features a constant labor share, which Kaldor described as having "a remarkable constancy in 'developed' capitalist economies" (p. 591). It is even more remarkable that this constancy appeared to hold well after Kaldor's paper: Prescott (1986) first for the United States, and Gollin (2002) later for many countries, found that the labor share had been roughly constant and close to 0.65-0.70. However, Karabarbounis and Neiman (2014, henceforth KN) recently document that the labor share has been slowly declining since the 1980s — precisely around the time that Gollin's analysis finishes. Following this, Giandrea and Sprague (2017) argue that the decline is more pronounced after 1997, and Koh et al. (2016), using a revised dataset, date the beginning of the decline to the late 1940s. While the literature has provided a number of exciting theories to explain the fall in the labor share (which we review below), this paper focuses squarely on how this decline affects the NGM's ability to reproduce macroeconomic aggregates. We compare the benchmark NGM with a constant labor share against versions of the model that feed in its observed decline. We find that the NGM with the constant labor share is preferred.

In the NGM, the labor share is governed by parameter α appearing in the exponents of the Cobb-Douglas production function.[2] In the benchmark model, this α is constant across periods which, as discussed, appears to be at odds with the data. While one could enhance the NGM by using a vector $\alpha = (\alpha_1, \alpha_2, ..., \alpha_T)$ to perfectly replicate the evolution of the labor share, it is unclear whether the model performance with respect to macroeconomic aggregates would be improved. Further, this enhanced version comes at the cost of $T - 1$ additional parameters, making the cross-model comparison uneven.

Statisticians commonly confront model comparison problems similar to the one posed above. They acknowledge that such a comparison is inherently subjective and as a result use

[1] This first sentence is almost verbatim the first sentence of Kehoe and Prescott (2007).

[2] The assumption of a competitive labor market makes the wage equal the marginal product of labor, $w = \partial Y/\partial L$. A Cobb-Douglas production function, $Y = AK^{\alpha}L^{1-\alpha}$, makes the marginal product of labor be $\partial Y/\partial L = (1-\alpha)Y/L$. Combining them both, we get that the labor share, wL/Y, is constant and equal to $1 - \alpha$.

rules of thumb, or "criteria," that reward a model's accuracy while penalizing the number of parameters employed. The most commonly used criteria are the Akaike Information Criteria (Akaike, 1973, henceforth AIC) and the Bayesian Information Criteria (Schwarz, 1978, henceforth BIC), which we use in our analysis to compare the NGM with a constant labor share against versions of the model where the labor share declines.[3] We compute the AIC and BIC for each of six different variables (wage, rental rate, investment, consumption, capital and hours worked) and find that the model with a constant labor share is always preferred.

In Section 2 we sketch our version of the NGM, provide a brief discussion of how to compute the different data series, how we calibrate the model, and explicitly write down the formulae for the two criteria. This section is deliberately short because our model, data, and calibration follow Conesa, Kehoe, and Ruhl (2007, henceforth referred to as CKR), and the criteria are already described in Akaike (1973) and Schwarz (1978).

In Section 3 we compare the benchmark NGM with four variants of the model. The first one linearly connects the labor shares of 1980 and 2014; the second one uses the fitted values of regressing the labor share against time; the third one, following the break point of Giandrea and Sprague (2017), has a constant labor share until 1997 and linearly declines afterwards; last, the fourth one has the exact evolution of the labor share. All versions share initial capital levels, population growth, preference parameters and depreciation rates because all of these values are calibrated independently of α. Then, for each version's path of α, we compute TFP and its implied long run growth for the sample period 1970-2014. Given these parameters, we simulate paths of wages, rental rates, investment, consumption, capital and hours worked, and compute the AIC and BIC. We find that the version with a constant labor share is always selected.

In Section 4 we perform a number of robustness checks. A particularly striking result from our model comparison is that investment and capital are better predicted by the version with a constant labor share than by any other. The versions with a declining labor share fare particularly worse following the Great Recession, raising the concern that the post-recession period is responsible for our findings. Excluding the years after 2007, we find that our results do not change.

During the time period of our analysis, the fall in the labor share has coincided with a fall in the relative price of investment. In a second robustness check, we build a two-sector

[3] In their general form, both criteria require knowledge of the number of parameters, the number of observations, and the model's likelihood — the last one, an object that our deterministic versions of the NGM do not have. However, when the model analyzed is the classical linear regression model with Gaussian errors, the likelihood term turns into the standard deviation of errors for both the AIC and BIC, which are the versions that we use.

version of the NGM and impose the observed decline in the relative price of investment. If we maintain our assumption of a Cobb-Douglas production function, we find that a constant labor share is still preferred.

The assumption of a Cobb-Douglas production function may be unduly restrictive. KN use a constant elasticity of substitution (CES) production function, which endogenizes the labor share in the model. They show that 50% of the fall in the labor share can be accounted for by the observed decline in the relative price of investment. To replicate our model comparison within the KN environment, we compute the series of weights in the CES function that account for the remaining 50% decline in the labor share. Then we compare the version with constant weights (a simplified version of the model in KN) with versions featuring time-varying weights. Again, we find that the model with constant weights is preferred.

Another concern with our model arises from our stylized use of a constant depreciation rate — standard practice when building the NGM, but also at odds with measured depreciation rates in the data. Employing a calibrated vector of time-varying depreciation rates (as opposed to a constant parameter) does not alter the results.

When we compute the NGM, we need to impose a constant TFP growth rate for the period when data is no longer available. Following standard practice, we set this long-run growth rate to equal the average growth rate of TFP for the period where data is available. Since different labor shares imply different series for TFP, each version of the NGM is computed using a different growth rate of TFP. We re-do the analysis imposing the same growth rate of TFP for all the versions of the model and find that the results do not change.

Throughout the analysis, the labor share is computed following CKR, which means that our series is (similar but) not exactly the same as in KN. We perform our benchmark comparison using the KN time series and find our results do not change.

Last, we also expand the set of countries in the analysis, repeating the comparison for all other G-7 economies (Canada, France, Germany, Italy, Japan, and the U.K.) and find that the NGM with a constant labor share largely remains the preferred one.

Literature review

KN are the first to document a decline in the labor share, and they argue that half of their measured decline can be accounted for by a decrease in the price of investment goods together with an elasticity of capital to labor larger than one. Since then, a number of alternative hypotheses have been proposed to explain this decline. We briefly review them here.

Elsby et al. (2013) claim that a third of the decline appears to be an artefact of mismeasurement and propose the offshoring of labor-intensive components in the U.S. supply chain as their preferred hypothesis to explain the decline. Koh et al. (2016), using a revised version of the National Income and Product Accounts (NIPA), argue that the entire decline in the U.S. labor share can be accounted for by intellectual property products capital. Glover and Short (2017) emphasize that the demographic composition of the labor force can account for roughly half of the decline in the labor share. Recently, Autor et al. (2017) propose that a few, low-labor share, large market share, superstar firms are responsible for the decline. Finally, Grossman et al. (2017) explore the possibility that the global productivity slowdown has directly led to a fall in the labor share.

Our paper has a different focus than the studies mentioned above: we do not propose a new hypothesis as to why the labor share declines. Instead, we take the decline as given, and simply ask whether explicitly accounting for this decline in the NGM can improve the model's performance. Our answer is relevant not only for the standard NGM as we have narrowly defined it, but to a broad swath of quantitative macroeconomics whose models build on the NGM with a constant labor share. Our finding provides initial evidence that a constant labor share may in fact be preferable for the study of the macroeconomic aggregates.

2 Model, data and criteria

In this section we provide all the tools we use in this paper. First, we quickly go over the neoclassical growth model, then we briefly discuss how we construct the data series, and finally explicitly write the criteria that we employ to evaluate different models.

2.1 The Neoclassical Growth Model

The version of the NGM we use in this paper is the same as in CKR. It consists of a representative household, endowed with initial capital K_{1980}, hours per year, $\overline{h}$, and total working age population, N_t, that takes the prices of labor, w_t, and rental rate of capital, r_t, as given, and chooses how much to consume, C_t, work, L_t, and invest (in the form of next period's capital), K_{t+1}, to maximize her discounted flow of utility,

$$\max_{\{C_t,L_t,K_{t+1}\}_{t=1980}^{\infty}} \sum_{t=1980}^{\infty} \beta^t \left(\gamma \log(C_t) + (1-\gamma)\log(\overline{h}N_t - L_t)\right) \tag{1}$$

such that $\forall t \geq 1980$

$$C_t + K_{t+1} \leq w_t L_t + (1-\delta+r_t)K_t, \tag{2}$$

$$C_t, K_{t+1}, L_t \geq 0, L_t \leq \overline{h}N_t, \tag{3}$$

where γ governs the relative weight of consumption over leisure, β is the discount factor, and δ is the depreciation rate of capital. There is a perfectly competitive, representative firm that hires labor and rents capital to operate a Cobb-Douglas production function, $Y_t = A_t K_t^{\alpha_t} L_t^{1-\alpha_t}$, where A_t is an exogenous productivity, implying that the wage and rental rate are given by

$$w_t = (1 - \alpha_t) A_t K_t^{\alpha_t} L_t^{-\alpha_t}, \tag{4}$$

$$r_t = \alpha_t A_t K_t^{\alpha_t - 1} L_t^{1-\alpha_t}. \tag{5}$$

Last, feasibility is satisfied,

$$C_t + K_{t+1} = Y_t + (1 - \delta) K_t. \tag{6}$$

Next, we show how to map this model to the data, in order to evaluate different versions of the model.

2.2 Data

To compare the different versions of the NGM, we consider a variety of calibration strategies that differ in their treatment of the sequence of α_t. We then forecast paths of consumption, C_t, investment, I_t, capital, K_t, hours worked, L_t, wages, w_t, and interest rates, r_t. Hence, to assess whether the predicted values for these variables are close to their empirical counterparts, we first need to construct the latter. Throughout, we add the superscript d when we refer to the values of a variable when taken from the *data*.

For the NGM to predict these six variables, on top of a calibrated (sequence of) α, it needs parameters β, δ, γ, K_{1970}, $\{N_t\}_{t=1980}^{2014}$, $\{A_t\}_{t=1980}^{2014}$, and long-run growth rates of population and TFP, g_N and g_A to which the economy converges. We next show how to construct them (although the steps are the same as CKR) using data from the OECD. It is important to stress that the only variables whose values depend on the calibrated (sequence of) α, are $\{A_t\}_{t=1980}^{2014}$ and g_A.

To compute the empirical hours worked, L_t^d, we simply multiply the number of workers by average hours worked. The sequence of working age population, N_t^d, is taken directly from the data, and its long-run growth rate, $g_N^d = 0.0045$ is the growth rate of the last year, 2013 to 2014.

We next compute (deflated) components of GDP. All variables taken directly from the National Accounts are expressed in current prices and deflated using the GDP deflator. The only exception is GDP itself, which we take from the National Accounts in both constant prices, $Y_t^{d,c}$, and nominal prices, $Y_t^{d,n}$, to construct the GDP deflator. We use Gross Capital Formation, GCF_t^d, to construct the deflated series for investment, $I_t^d = GCF_t^d \times Y_t^{d,c} / Y_t^{d,n}$.

Since the model is a closed economy with no government, the series for consumption is then $C_t^d = Y_t^{d,c} - I_t^d$.

The series of Consumption of Fixed Capital, CFC_t^d, is used to compute the depreciation rate, δ, according to

$$\frac{1}{34}\sum_{t=1981}^{2014}\frac{\text{CFC}_t^d}{Y_t^{d,n}} = \frac{1}{10}\sum_{t=1971}^{1980}\frac{\delta K_t^d}{Y_t^{d,c}}, \tag{7}$$

where the series for aggregate capital, K_t^d, is constructed using the perpetual inventory method,

$$K_{t+1}^d = (1-\delta)K_t^d + I_t^d, \tag{8}$$

and K_{1970}^d satisfies

$$\frac{K_{1970}^d}{Y_{1970}^{d,c}} = \frac{1}{10}\sum_{t=1971}^{1980}\frac{K_t^d}{Y_t^{d,c}}. \tag{9}$$

These equations imply $\delta = 0.0677$ and $K_{1970}^d = 2.25Y_{1970}^{d,c}$.[4]

We next move to the computation of the labor share and rental prices. If the data were perfectly consistent with the model outlined in the previous section, we would compute period t's labor share as the fraction of GDP used for compensation of employees, CE_t^d. Namely,

$$1-\alpha_t^d = \frac{CE_t^d}{Y_t^{d,n}}.$$

Then, the empirical wage would be

$$w_t^d = \frac{(1-\alpha_t^d)Y_t^{d,c}}{L_t^d}, \tag{10}$$

and the rental rate

$$r_t^d = \frac{\alpha_t^d Y_t^{d,c}}{K_t^d}. \tag{11}$$

However, GDP at market prices includes indirect taxation, T_t^d, on top of payments to labor and capital. Moreover, the Mixed Income payments from the Household sector, $MI_t^d(HH)$, include a large proportion of payments to labor services provided by the business owner and her family (see CKR for details). Due to these two issues, a more appropriate measure of the labor share is

$$1-\alpha_t^d = \frac{CE_t^d - CE_t^d(HH)}{Y_t^{d,n} - CE_t^d(HH) - MI_t^d(HH) - T_t^d}, \tag{12}$$

[4]Equation (9) is the right way to compute initial capital if the economy is close to a balanced growth path at the beginning of the time period. We believe that the assumption of a balanced growth path already in 1970 is sensible: the initial capital-output ratio, 2.25, is in the middle of the values we get for the following decade of 2.14 and 2.31, and close to the average between 1970-2014 of 2.28.

where $CE_t^d(HH)$ is compensation of employees from the Household sector. Then, the wage and the rental rate of capital are computed using their definitions, equations (10) and (11), using the series for α_t^d from equation (12).

Note that the Euler equation and the labor supply equation that arise from maximizing equation (1) subject to equation (2) are given by

$$\frac{1}{C_t} = \beta(1 - \delta + r_{t+1}) \times \frac{1}{C_{t+1}}, \tag{13}$$

and

$$\frac{\gamma}{C_t} = \frac{1-\gamma}{w_t(\bar{h}N_t - L_t)}. \tag{14}$$

We use the first expression to solve for β, the second one to solve for γ, and then calibrate the two parameters by simply averaging for the period 1970-1979. Namely,

$$\beta = \frac{1}{10}\sum_{t=1970}^{1979} \frac{1}{1-\delta + r_{t+1}^d} \times \frac{C_{t+1}^d}{C_t^d} = 0.9694, \tag{15}$$

and

$$\gamma = \frac{1}{10}\sum_{t=1970}^{1979} \frac{C_t^d}{w_t^d(\bar{h}N_t^d - L_t^d) + C_t^d} = 0.2562. \tag{16}$$

Note that the calibration strategy thus far depends only on empirical series. Hence, it will be used across all versions of the NGM that we analyze in the paper. Calibration of the remaining parameters, however, will depend on the particular version being considered.

We analyze five different versions of the NGM. In the first, *Constant*, the labor share is constant and equal to its average during the period 1980-2014. Namely,

$$1 - \alpha^{cons} = 1 - \sum_{t=1980}^{2014} \alpha_t^d / 34 = 0.6913. \tag{17}$$

The second, *Trend*, features a labor share that linearly connects the first and last terms of the $1 - \alpha_t^d$ series. Particularly,

$$1 - \alpha_t^{trend} = 1 - \left(\alpha_{1980}^d + \frac{\alpha_{2014}^d - \alpha_{1980}^d}{34} t\right) = 0.7037 - 0.001068(t - 1980). \tag{18}$$

The third, *Regression*, derives a labor share series from the fitted values that results from regressing α_t^d against time t. Specifically,

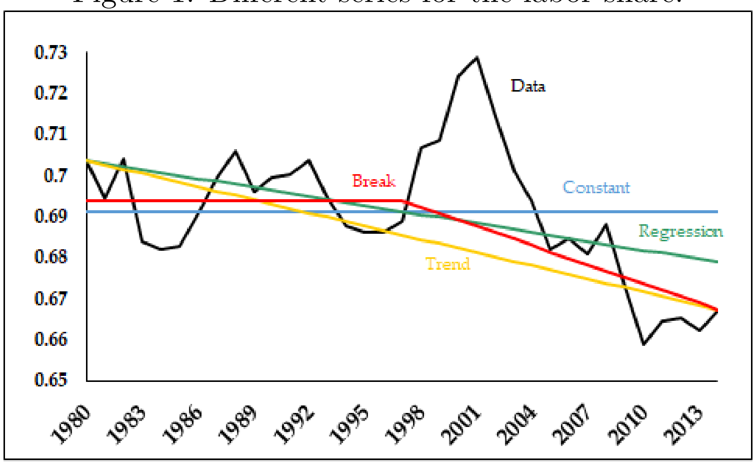
$$1 - \alpha_t^{reg} = 1 - \beta_{0,reg} - \beta_{1,reg} t = 0.7037 - 0.00073(t - 1980), \tag{19}$$

where $\beta_{0,reg}$ and $\beta_{1,reg}$ are the coefficients of the linear regression.[5] The fourth, *Break*, features a labor share that is constant up to 1997, and declines linearly thereafter (the year 1997 is chosen to coincide with the break point proposed by Giandrea and Sprague, 2017). That is,

$$1-\alpha_t^{break}=\begin{cases}1-\sum_{t=1980}^{1997}\alpha_t^d/17 & \text{if } t\leq 1997\\ 1-\alpha_{1997}^{break}-\dfrac{\alpha_{2014}^d-\alpha_{1997}^{break}}{17}(t-1997) & \text{if } t>1997.\end{cases} \tag{20}$$

Finally, the fifth, *Exact*, feeds in the labor share from the data, equation (12). We plot the different calibrated series of the labor share in Figure 1.

Figure 1: Different series for the labor share.



TFP is computed as a residual that depends on series of capital, labor, GDP and calibrated α. Namely, A_t^i, is a function of the calibrated α_t^i, $i\in\{cons, trend, reg, break, exact\}$, and given by

$$A_t^i=\frac{Y_t^{d,c}}{(K_t^d)^{\alpha_t^i}(L_t^d)^{1-\alpha_t^i}}. \tag{21}$$

Computing the model requires a growth rate of TFP after year 2014, which we refer to as g_A^i, and set equal to the average growth rate of A_t^i during the period 1980-2014.[6] Every exercise jointly uses α_t^i, A_t^i and g_A^i in simulating the model.

It is important to stress that each version of the model consists of a vector α^i together with a vector A^i — that is, we cannot separately calibrate a TFP series from the series of α. To see why, suppose that A^i was the same in all the versions (and, for the sake of argument, it was the one derived from having a constant parameter, α_{cons}, making it A^{cons}).

[5] The fact that α_{1980}^{trend} and α_{1980}^{reg} are both equal to 0.7037 is a pure coincidence.

[6] The values of this growth rate are $g_A^{cons}=1.085\%$, $g_A^{trend}=0.572\%$, $g_A^{reg}=0.741\%$, $g_A^{break}=0.699\%$, and $g_A^{exact}=0.572\%$.

Then, suppose there was a different version of the model, version i, that perfectly predicted the paths of capital, $K_t^i = K_t^d$, and labor, $L_t^i = L_t^d$, while having a different path for α, $\alpha^i \neq \alpha^{cons}$. Then, this version i would wrongly predict aggregate output,

$$\begin{aligned} Y_t^i &= A_t^{cons}(K_t^d)^{\alpha_t^i}(L_t^d)^{1-\alpha_t^i} = \frac{Y_t^{d,c}}{(K_t^d)^{\alpha_t^C}(L_t^d)^{1-\alpha_t^C}}(K_t^d)^{\alpha_t^i}(L_t^d)^{1-\alpha_t^i} \\ &= Y_t^{d,c}(K_t^d)^{\alpha_t^i-\alpha_t^{cons}}(L_t^d)^{\alpha_t^{cons}-\alpha_t^i} \neq Y_t^{d,c}. \end{aligned}$$

Using a similar argument, the predicted paths for wages, w_t, and rental rates, r_t, would be off. Moreover, a wrong prediction of aggregate output also implies that either the series of consumption or the series of investment (or both) are wrong, because $Y_t^i = C_t^i + I_t^i$.

2.3 Criteria

We compare the different versions of the NGM using two well-known criteria developed in Statistics. These are the Akaike Information Criteria (Akaike, 1973, henceforth AIC) and the Bayesian Information Criteria (Schwarz, 1978, henceforth BIC), which are calculated using the number of parameters in the model, p, the model's likelihood, $\mathcal{L}$, and the number of observations, n, according to

$$AIC = 2 \times p - 2 \times \log(\mathcal{L}),$$

and

$$BIC = \log(n) \times p - 2 \times \log(\mathcal{L}).$$

To keep our comparison as close as possible to common calibration practice in the macroeconomics literature, we compute deterministic paths for each version of the model. To compare different model predictions, we can use the AIC and BIC versions for the classic linear regression model with Gaussian errors,

$$AIC = 2 \times n \times \log \sigma + 2 \times (p+1) + n \times (1 + \log(2\pi)), \tag{22}$$

and

$$BIC = 2 \times n \times \log \sigma + \log(n) \times (p+1) + n \times (1 + \log(2\pi)), \tag{23}$$

which are functions of the standard deviation of the errors, σ, a number easy to compute with model predictions and data. In particular, the standard deviation of the errors of variable

$\{x_t^i\}_{t=1980}^{2014}$, for version of the model $i \in \{cons, trend, reg, break, exact\}$, is

$$\sigma^i(x) = \frac{1}{N}\sqrt{\sum_{t=1980}^{2014}(x_t^i - x_t^d)^2}, \tag{24}$$

where x stands for wage w, interest rate r, consumption C, investment I, capital K, and hours worked L, and $\{x_t^d\}_{t=1980}^{2014}$ is the variable's empirical counterpart.

Then, the cross-model comparison using variable x and criteria $CIC = \{AIC, BIC\}$ is done by computing $CIC^i(x)$ for all $i \in \{cons, trend, reg, break, exact\}$ and selecting the one with the smallest value. In total, we compute twelve criteria (two criteria for each of the six different variables) for each version of the model. In the following section we show that the version with a constant labor share is selected all twelve times.

3 Model comparison

In this section we simulate the five versions of the NGM that we consider (*Constant, Trend, Regression, Break and Exact*) and use the paths of r, w, I, C, K and L, together with their empirical counterparts, to compute the AIC and BIC scores.[7] In each version, the number of observations is always 35 (the number of annual observations between 1980 and 2014). The different versions also share the following number of parameters: 35 TFPs, A_t^i, 35 labor endowments, N_t, the number of hours that can be worked in a year, $\bar{h}$, initial capital, K_{1980}, depreciation rate, δ, discount factor, β, disutility of work, γ, long-run population and TFP growth rates, g_N and g_A^i, for a total of 77 parameters. Moreover, in *Constant* we have 1 additional parameter for the labor share; in *Trend* and *Regression* we have 3 additional parameters (initial labor share, slope, and final labor share through the balanced growth path); in *Break* we have 4 additional parameters (initial labor share, period when it starts to decline, slope, and final labor share); and in *Exact* we have 35 additional parameters (one for each period for which we have observations). Therefore, in total there are 78, 80, 80, 81, and 112 parameters in the different versions of the NGM we simulate.

We find that regardless of the variable chosen or the criteria employed, *Constant* is always selected — it always has the smallest score. Scores across criteria and variables are reported in Table 1.

To see the role of penalties in our analysis, in what follows we compare *Constant* to the remaining versions of the model. Within a given variable, differences in scores reflect the

[7] To compute the numerical solution of the model, we modify the Matlab codes developed by Kim Ruhl for CKR. We use the last version available, from April 2008, which can be freely downloaded from greatdepressionsbook.com.

Table 1: AIC and BIC, NGMs

AIC	r	w	I	C	K	L
Constant	81.16	3.85	1026.36	1052.62	1068.55	732.31
Trend	85.26	10.47	1049.61	1056.97	1097.63	733.39
Regression	85.62	9.51	1041.62	1057.04	1086.55	734.40
Break	86.66	11.19	1051.30	1059.32	1098.10	735.46
Exact	145.31	73.67	1117.81	1121.86	1163.29	798.74
BIC	r	w	I	C	K	L
Constant	45.14	-32.17	990.34	1016.60	1032.53	696.29
Trend	48.33	-26.47	1012.68	1020.04	1060.70	696.86
Regression	48.69	-27.42	1004.69	1020.10	1049.62	697.47
Break	49.27	-26.20	1013.92	1021.93	1060.71	698.07
Exact	93.79	22.15	1066.29	1070.34	1111.77	747.22

sum of differences in the standard deviation of errors and the additional penalty received due to the extra number of parameters employed. In the case of the AIC, the difference in penalties with respect to *Constant* is simply two times the number of additional parameters, see equation (22). Hence, if the difference in scores is larger than the difference in penalties, *Constant* is preferred even if penalties are not introduced.

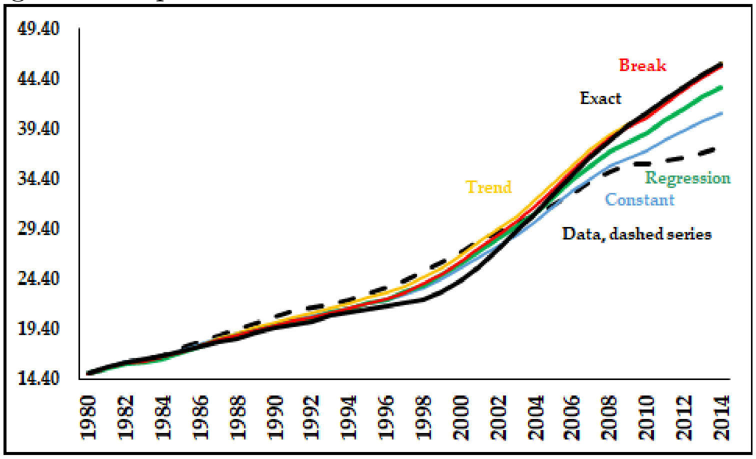
While *Constant* has a smaller score than *Trend, Regression, Break* and *Exact* for all variables, the difference is largest for investment and capital. Consider, for instance, *Trend* and *Regression*. In them, the penalty when using the AIC is 4 units higher than in *Constant*. However, the differences in scores are 15.26 and 23.25 for I (1026.36 vs. 1041.62 and 1049.61) and 20.00 and 30.08 for K (1068.55 vs. 1086.55 and 1097.63). This implies that *Constant* is preferred even in the absence of penalties. The same is true for the other versions of the model: the penalty difference between *Break* and *Constant* is 6 units; again, much smaller than the 24.94 and 29.55 differences for I and K. Last, the penalty difference between *Exact* and *Constant* is 68 units; again, smaller than the 91.45 and 94.74 difference in scores.

The reason for this finding is that *Trend, Regression, Break* and *Exact*, all imply a final α_{2014} that is substantially larger than *Constant*. In turn, a larger α implies a greater return to capital, which induces further capital accumulation by the household. Since *Constant* already over-predicts the capital stock, all other versions make even worse predictions. For a visual illustration of how models predict the capital stock, in Figure 2 we plot the series of capital from the data and predicted by each version of the model.

To put the magnitudes of Figure 2 in perspective, the difference in capital stocks between *Trend* and *Constant* equals 31.2% of 2014's actual GDP. This figure is 15.8%, 29.0% and 30.7% for *Regression, Break* and *Exact*.[8]

[8]The overprediction of capital stock by all models is particularly important in the last decade of the

Figure 2: Capital series. Units in Trillions of 2010 dollars.



There are two variables, wage and consumption, for which the difference between AIC scores is (larger but) similar to the difference in penalties. *Trend* and *Regression* versions have scores that are 6.62 and 5.66 units larger for the wage, and 4.35 and 4.42 units larger for consumption (with a penalty difference of 4). *Break* has scores that are 7.34 units larger for the wage, and 6.70 units larger for consumption (with a penalty difference of 6). Last, *Exact* has scores that are 69.82 units larger for the wage and 69.24 units larger for consumption (with a penalty difference of 68). This means that the extra parameters used by these models do not yield better predictions than for *Constant*, but these predictions are not substantially worse either (as they were in the case of investment and capital). Model comparisons for interest rates are similar, with some versions performing better than *Constant* and some worse once penalties are removed. *Trend* and *Regression* have differences in score of 4.10 and 4.48 units (with a penalty difference of 4), whereas *Break* has a difference of 5.50 and *Exact* of 64.15 (with penalty differences of 6 and 68).

The last variable, hours worked, is different. Although *Constant* is selected when using both the AIC and BIC, all other versions perform better in the absence of penalties. The differences in scores are 1.08 (*Trend*), 2.09 (*Regression*), 3.15 (*Break*), and 66.43 (*Exact*), all of them below the penalty differences of 4, 4, 6, and 68. This result — accurate movements in the labor share parameter can improve the predictive ability of the model — is in line with the findings of Santaeulalia and Rios-Rull (2010).

Our analysis draws a very stark picture: *Constant* is always selected over *Trend, Regression, Break* and *Exact.* We interpret this result as saying that, despite the fall in the labor share observed in the data, the constant labor share version of the NGM still does a very

sample. In the following section we discuss the role of that decade in our analysis, in the sub-section referred to as "Removing the Great Recession period."

good job at predicting macroeconomic aggregates. In the following Section we perform a number of robustness checks and find that our conclusion remains unchanged.

4 Robustness checks

There are a number of factors that may be driving our results in the previous section. For instance, the time period we cover includes the Great Recession, a period of economic turmoil. Also, the fall in the labor share coincides with a period of decreasing investment prices, as already noted in KN. Moreover, in the model we assume a constant depreciation rate, when in the data it fluctuates. Additionally, in order to solve the model we impose a long-run growth rate of TFP that, by construction, affect the results. Furthermore, the series for the labor share that we use are not the same that KN use, and the results are only derived for one country, the United States. In this Section we explore whether these issues affect our conclusions, and find that they do not.

4.1 Removing the Great Recession period

The Great Recession occurred in the last years of our analysis. As can be seen in Figure 2, while none of the versions of the NGM does particularly well after that event, *Constant* happens to perform substantially better than the rest. Given the economic turmoil of the time however, it is unclear how much this improved performance should be viewed as support for *Constant* rather than mere coincidence. To check whether our results are driven by this extraordinary time period, Table 2 reproduces Table 1 for the period 1980-2006 only.

Table 2: Removing the Great Recession from analysis.

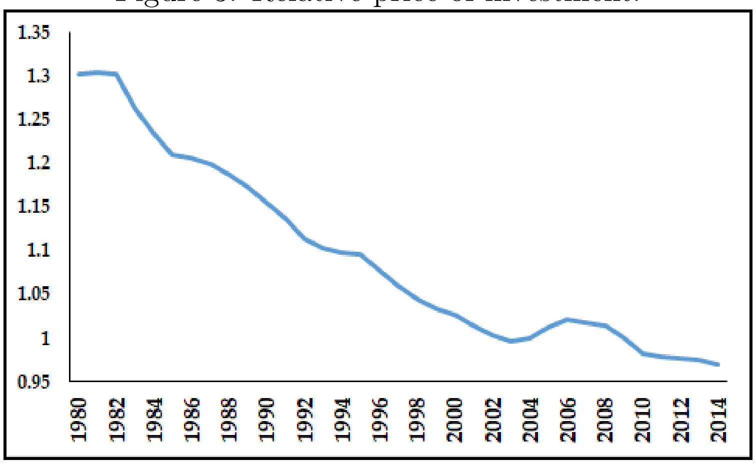
AIC	r	w	I	C	K	L
Constant	69.60	-2.60	1010.28	1053.14	1058.04	734.94
Trend	75.18	4.06	1030.24	1057.60	1059.37	736.49
Regression	75.57	3.85	1022.69	1057.56	1059.66	737.08
Break	76.29	4.72	1032.58	1059.84	1063.69	738.53
Exact	131.87	68.78	1105.79	1122.07	1141.53	801.96
BIC	r	w	I	C	K	L
Constant	24.68	-47.53	965.36	1008.22	1013.12	690.02
Trend	29.13	-42.00	984.18	1011.54	1013.31	690.43
Regression	29.51	-42.21	976.63	1011.50	1013.60	691.02
Break	29.66	-41.90	985.95	1013.21	1017.06	691.90
Exact	67.61	4.52	1041.53	1057.82	1077.27	737.71

A quick inspection of Table 2 shows that results do not change: *Constant* is still selected in every comparison. However, its relative performance is diminished when looking at capital. Now, *Constant* is only barely preferred, as differences in scores are smaller than differences in penalties. While excluding the Great Recession does not change our results, it does reduce the preeminence of *Constant* over the alternatives.

4.2 Falling price of investment

The period we study coincides with a secular decline in the relative price of investment, see Figure 3. In this robustness exercise we check whether our results change if our model is expanded to feature this decline.

Figure 3: Relative price of investment.



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The model in Section 2 has only one sector, implying that the relative price of investment is constant and equal to one. To overcome this, we build a (well-known) two-sector growth model, where household's investment, I_t, is transformed into new capital. This transformation is done by the investment sector, whose productivity $Z_{I,t}$ can change over time. Compared to the model we have used so far, equation (2) turns into

$$C_t + I_t \leq w_t L_t + r_t K_t, \tag{25}$$

where

$$K_{t+1} - (1-\delta)K_t = Z_{I,t} I_t. \tag{26}$$

With $Z_{I,t}$ growing over time, the relative price of investment becomes cheaper. Other than this change, the model we employ is the same that we have been using throughout — in particular, we still assume that the production function is Cobb-Douglas (in the exercise that follows we change this assumption). We generate predictions by feeding the model with series $Z_{I,t}$ equal to the inverse of the relative price of investment.

We solve the different versions of the NGM — *Constant, Trend, Regression, Break*, and *Exact* — generate series for the macroeconomic variables that we are interested in — wage, interest rate, investment, consumption, capital, and hours worked — and compute the AIC and BIC for all of them. The results are in Table 3.

Table 3: Falling price of investment.

AIC	r	w	I	C	K	L
Constant	167.73	79.97	1107.32	1112.35	1143.34	787.03
Trend	170.71	85.67	1123.32	1116.55	1165.83	788.89
Regression	171.02	84.44	1117.34	1116.59	1157.43	789.20
Break	172.96	87.06	1124.71	1118.85	1166.22	790.77
Exact	232.62	149.33	1189.06	1181.11	1229.29	854.99
BIC	r	w	I	C	K	L
Constant	115.75	27.99	1055.35	1060.37	1091.36	735.06
Trend	117.82	32.78	1070.43	1063.66	1112.94	736.00
Regression	118.13	31.56	1064.45	1063.70	1104.54	736.31
Break	119.62	33.71	1071.37	1065.50	1112.87	737.43
Exact	165.14	81.85	1121.58	1113.63	1161.81	787.52

The main conclusion that we draw from this robustness exercise is that, again, *Constant* is selected over all the alternatives — regardless of the variable that we look at, and regardless of the criteria employed. Similar to what happened in the benchmark exercise, *Constant* does particularly better when predicting I and K. An interesting feature of this second exercise, though, is that the magnitude of these differences have shrunk. All versions improve their predicted paths of the capital stock, with the worst performing versions in the benchmark comparison of Section 3 seeing the largest gains (even though they are still not selected).

4.3 CES production function

In the previous exercise, the evolution of the price of investment and the labor share are orthogonal to one another. But as KN show, when the production technology features a capital-labor elasticity of substitution larger than one, half of the decline in the labor share can be explained by the fall in the price of investment.

Building on that paper, in this robustness exercise we incorporate a constant elasticity of subsitution (CES) production function to the model of Section 4.2. Namely, we set output to be

$$Y_t = \left(\alpha_t K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha_t)\left(E_t L_t\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where α_t is the time-varying weight of capital in final output, σ is the elasticity of substitution between capital and labor (following KN, we set it to 1.25), and E_t is the labor-augmenting productivity. In this model, the labor share is given by

$$\frac{w_t L_t}{Y_t} = (1-\alpha_t) \frac{(E_t L_t)^{\frac{\sigma-1}{\sigma}}}{\alpha_t K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha_t)(E_t L_t)^{\frac{\sigma-1}{\sigma}}}.$$

Note that if the elasticity of substitution is one we obtain the Cobb-Douglas case, with E_t equal to $A_t^{\frac{1}{1-\alpha}}$, and the labor share would one minus the capital weight, $1-\alpha_t$.

This CES production function requires calibrating a new series for α_t^d from the data. As before, *Constant* features a constant parameter α^{cons} equal to the average of the series from the data. The series for α_t^{trend}, α_t^{reg}, α_t^{break} and α_t^{exact} are also recomputed accordingly.

With this production function, the labor share is not going to be constant for any version of the model: the falling price of investment makes K_t grow more over time than $E_t L_t$, which implies that the labor share of all versions, including *Constant*, falls over time.

As before, we simulate the model output for each of the versions, and compute the AIC and BIC. The results can be seen in Table 4.

Table 4: CES production function and falling price of investment.

AIC	r	w	I	C	K	L
Constant	169.10	190.17	1118.93	1114.72	1160.04	787.66
Trend	196.20	175.94	1131.95	1183.37	1186.87	803.15
Regression	197.75	175.23	1135.67	1184.30	1190.33	804.03
Break	174.61	196.20	1134.29	1121.02	1177.84	792.34
Exact	233.71	258.17	1196.81	1183.31	1237.94	858.11
BIC	r	w	I	C	K	L
Constant	116.67	137.37	1066.50	1062.29	1107.60	735.23
Trend	142.85	122.60	1078.61	1130.03	1133.53	749.81
Regression	144.41	121.89	1082.33	1130.96	1136.99	750.69
Break	120.81	142.40	1080.49	1067.22	1124.04	738.54
Exact	165.77	190.24	1128.88	1115.38	1170.01	790.18

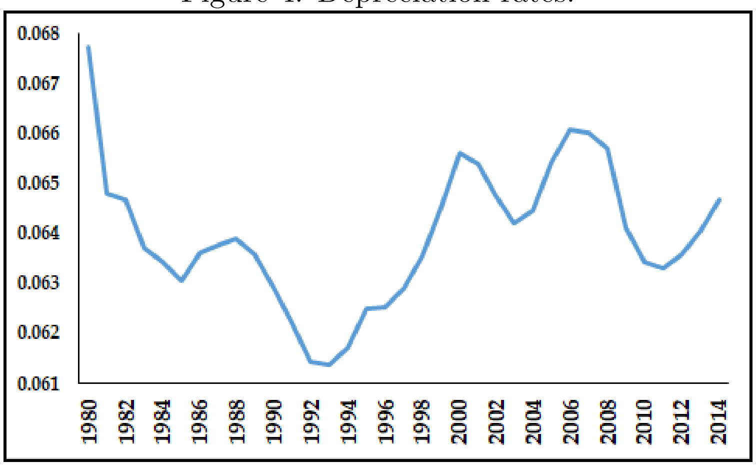
The results point in the same direction as our benchmark comparison: *Constant* is preferred over all other versions of the model in almost all variables with the exception of wages, where *Regression* is preferred (and *Trend* is nearly as good). Still, the results clearly indicate an overall preference for the model with a constant capital weight, α. our interpretation of the results is that the model with a constant labor parameter is selected. Namely, the KN model is selected over versions of the model with time-varying capital weights in the production function.

4.4 Time-varying depreciation rate

In this next exercise we evaluate whether the assumption of a constant depreciation rate is affecting our results. In the benchmark comparison, where *Constant* is always selected, we have seen that the four other models performed particularly poorly in predicted paths of capital. Clearly, the depreciation rate has a direct effect on the capital stock, see equation (6), raising the concern that the assumption of a constant depreciation rate may bias our cross-model comparisons.

In Figure 4, we plot the series for the depreciation rate that we obtain by simply allowing a vector — instead of a constant — to match the evolution of consumption of fixed capital as a fraction of GDP. The Figure clearly illustrates that the depreciation rate is not constant, fluctuating within the range of 0.061 and 0.068 throughout the period. To compute these

Figure 4: Depreciation rates.



series, we impose vectors δ_t and K_t^d be consistent with the series of consumption of fixed capital every period. Namely, equations (7) and (8) from Section 2 are replaced by

$$\frac{\text{CFC}_t}{Y_t} = \frac{\delta_t K_t^d}{Y_t}, \tag{27}$$

and

$$K_{t+1}^d = (1 - \delta_t) K_t^d + I_t^d. \tag{28}$$

We then solve the different versions of the model using this vector of time-varying depreciation rates, and compare their predictions against the data. Results are in Table 5.

The picture that emerges from this exercise is very similar to the benchmark: *Constant* is always selected — regardless of criteria employed or the variable we look at. In this robustness exercise, *Constant* still performs substantially better in predicting capital and

Table 5: Changing δ.

AIC	r	w	I	C	K	L
Constant	148.49	71.38	1094.23	1120.86	1136.07	801.21
Trend	152.43	77.81	1116.62	1125.25	1163.44	802.74
Regression	152.80	77.05	1108.67	1125.30	1152.00	803.33
Break	153.82	78.57	1118.41	1127.61	1164.01	804.63
Exact	212.35	141.02	1185.47	1190.18	1230.09	867.63
BIC	r	w	I	C	K	L
Constant	96.67	19.86	1042.23	1069.34	1084.55	749.69
Trend	100.00	25.38	1064.18	1072.82	1111.01	750.30
Regression	100.37	24.62	1056.24	1072.87	1099.57	750.89
Break	100.93	25.69	1065.52	1074.73	1111.12	751.74
Exact	145.32	74.00	1118.45	1123.15	1163.07	800.61

investment (which means that the time-varying depreciation rate affects all the different models similarly). As in the benchmark, all the models perform similarly when predicting hours worked.

4.5 Same long-run TFP growth rate

When we compute the solution of each model, we need to impose a long-run growth rate of TFP, g_A^i. As discussed in the calibration section of the model, we set that object to be the average growth rate of A_t^i for the period 1970-2014. Each version of the model has a different vector of α_t^i, which implies that each version has a different A_t^i, see equation (21). As a result, in our benchmark comparison each version of the model has a different long-run growth rate of TFP. Then, a natural concern is whether the success of *Constant* is driven by the labor share being constant or by different models having different parameter values for the long-run growth rate of TFP. Note that the latter is a completely ad-hoc assumption that can be changed without altering any other part of the exercise.

In this robustness exercise we solve all the versions of the model, imposing the same long-run growth rate to all of them, g_A^{cons}. We report the findings in Table 6.

As in the benchmark comparison, the model with a constant labor share is always selected, implying that the results are driven by the labor share being constant, rather than by the choice of g_A^i. However, the results also show that the concern was not ill-founded: all the versions fare better in this exercise than in Section 3.

Table 6: Same long-run TFP growth rate

AIC	r	w	I	C	K	L
Constant	81.16	3.85	1026.36	1052.62	1068.55	732.31
Trend	84.94	9.87	1045.91	1056.83	1095.01	734.24
Regression	85.41	9.13	1038.76	1056.95	1084.37	734.67
Break	86.41	10.73	1048.60	1059.22	1096.12	736.07
Exact	144.93	73.11	1114.99	1121.72	1161.08	799.14
BIC	r	w	I	C	K	L
Constant	45.14	-32.17	990.34	1016.60	1032.53	696.29
Trend	48.01	-27.06	1008.98	1019.90	1058.08	697.31
Regression	48.48	-27.80	1001.83	1020.02	1047.44	697.74
Break	49.03	-26.66	1011.21	1021.83	1058.73	698.69
Exact	93.41	21.59	1063.47	1070.20	1109.56	747.62

4.6 Labor shares of KN

Throughout our analysis, the labor share is computed directly from the National Accounts (and retrieved from Source OECD data) following the approach of CKR. Since KN follow a different approach to measuring it — and were the first to show its decline — we ask whether our results are changed with their labor share series. In Table 7 we report the results of repeating our exercise using their data.

Table 7: KN labor shares

AIC	r	w	I	C	K	L
Constant	99.58	44.56	1026.12	984.61	1086.35	700.74
Trend	101.88	48.96	1032.75	988.96	1092.11	703.32
Regression	101.18	50.48	1035.77	989.90	1094.49	702.28
Break	104.74	51.82	1037.00	991.95	1095.57	704.24
Exact	172.84	159.84	1077.37	1114.83	1142.31	764.64
BIC	r	w	I	C	K	L
Constant	63.47	8.45	990.00	948.50	1050.24	664.63
Trend	64.81	11.89	995.67	951.89	1055.03	666.24
Regression	64.11	13.40	998.70	952.83	1057.41	665.21
Break	67.18	14.27	999.44	954.39	1058.01	666.68
Exact	121.32	108.32	1025.85	1063.31	1090.79	713.12

We again find that *Constant* is preferred, regardless of the variable considered or the criteria employed. Hence, the results we find are robust to using these alternative series for the labor share.

4.7 Other countries

In the last robustness check we repeat the exercise from Section 3, this time performing the analysis for Canada, France, Germany, Italy, Japan, and the U.K. (all the remaining G-7 countries). Since we cannot construct the labor share for Canada the same way we did in the benchmark exercise for the United States (Conesa and Pujolas, 2017), we use the computed labor shares from KN for all six countries. As in the benchmark exercise, the picture that emerges is that *Constant* is typically (although not always) selected. All the results are reported in Appendix Tables: A.2 for the interest rate, A.3 for the wage, A.4 for investment, A.5 for consumption, A.6 for capital, and A.7 for hours worked. The tables contain the AIC and BIC scores for all countries and, for comparability purposes, also for the United States — whose figures are already in Table 7.

In this analysis there are six countries, six variables, and two criteria. Out of the seventy-two ($= 6 \times 6 \times 2$) different comparisons, *Constant* is selected sixty-one times. *Regression* is selected six times and *Trend* five. Neither *Break* nor *Exact* are ever selected.

Constant is always selected when the variable analyzed is the wage — regardless of the country or criteria. It is also selected for five out of six countries when the variables are the interest rate (all but U.K.), consumption (all but Germany) and hours worked (all but Italy) and the criteria is BIC — in these three cases, *Trend* is selected instead. When the criteria is the AIC, *Constant* is always selected for interest rate, consumption and hours worked.

Constant is also selected when the variables are investment and capital, and the countries are Germany, Italy and Japan. In the case of U.K., both criteria select *Regression* for both investment and capital. In the case of Canada, both criteria select *Regression* for investment, and *Trend* for capital.

If we include the United States in our summary, *Constant* is selected sixty-seven times out of seventy-eight comparisons (using KN measurements of the labor share and the G-7 countries). While not uniform dominance, we view these numbers as very strong evidence in favor of using the NGM with a constant labor share.

5 Conclusion

In this paper we ask whether forcing the Neoclassical Growth Model to explicitly account for the observed decline in the labor share improves its ability to reproduce macroeconomic aggregates. We find that it does not, which we view as very good news. Our findings hold for both a Cobb-Douglas and a Constant Elasticity of Substitution production function. Moreover a large number of robustness checks provide evidence that using constant parameters in these production functions is the right way to proceed.

However, even though we have tried to be as exhaustive as possible in the set of robustness checks performed, there will always be alternative exercises that may challenge our findings. We believe that future research will help clarify whether our results stand. If they do, we believe the neoclassical growth model with a constant labor share will remain the workhorse of modern macroeconomics.

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Appendix

Table A.2: G-7 countries, test based on r

AIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	100.46	100.60	122.43	115.46	112.87	116.75	99.58
Trend	104.19	103.43	126.41	120.11	117.79	117.32	101.88
Regression	103.73	103.09	125.78	119.38	116.32	118.78	101.18
Break	106.17	106.10	128.34	122.12	119.87	118.30	104.74
Exact	166.85	166.11	189.20	181.92	179.78	179.51	172.84
BIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	61.37	61.51	61.63	54.66	54.04	55.95	63.47
Trend	64.11	63.35	64.07	57.78	57.46	54.98	64.81
Regression	63.65	63.01	63.45	57.05	55.99	56.45	64.11
Break	65.59	65.52	65.24	59.01	58.81	55.20	67.18
Exact	110.94	110.19	102.24	94.96	95.63	92.55	121.32

Table A.3: G-7 countries, test based on w

AIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	28.22	40.37	92.93	72.97	156.45	73.59	44.56
Trend	33.29	43.50	98.14	77.88	162.32	77.40	48.96
Regression	33.26	44.30	97.77	77.25	161.09	77.43	50.48
Break	34.43	45.79	99.92	79.90	164.47	79.85	51.82
Exact	97.79	108.37	161.71	139.58	225.09	142.11	159.84
BIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	-10.87	1.27	32.13	12.18	97.62	12.80	8.45
Trend	-6.80	3.42	35.81	15.54	102.00	15.07	11.89
Regression	-6.82	4.21	35.43	14.92	100.76	15.10	13.40
Break	-6.15	5.21	36.82	16.79	103.41	16.75	14.27
Exact	41.88	52.45	74.76	52.62	140.93	55.15	108.32

Table A.4: G-7 countries, test based on I

AIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	894.95	932.37	569.37	555.59	685.40	538.01	1026.12
Trend	895.44	934.80	579.08	561.38	692.23	541.15	1032.75
Regression	894.69	940.45	581.35	561.10	694.88	536.83	1035.77
Break	897.42	936.65	580.86	563.39	694.25	545.27	1037.00
Exact	962.97	1000.68	643.94	626.39	756.45	605.24	1077.37
BIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	855.86	893.27	508.57	494.80	626.57	477.22	990.00
Trend	855.36	894.72	516.75	499.05	631.91	478.82	995.67
Regression	854.61	900.37	519.01	498.76	634.56	474.49	998.70
Break	856.85	896.08	517.76	500.29	633.18	482.17	999.44
Exact	907.05	944.76	556.98	539.43	672.30	518.29	1025.85

Table A.5: G-7 countries, test based on C

AIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	887.94	935.95	550.46	535.59	662.73	540.67	984.61
Trend	893.46	939.13	551.31	542.34	665.43	545.92	988.96
Regression	891.81	940.01	554.10	539.20	668.71	545.16	989.90
Break	895.85	941.39	554.11	544.44	667.12	548.71	991.95
Exact	955.14	1004.68	617.07	602.38	730.99	610.81	1114.83
BIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	848.84	896.86	489.67	474.79	603.90	479.87	948.50
Trend	853.37	899.05	488.98	480.01	605.11	483.59	951.89
Regression	851.73	899.93	491.76	476.87	608.38	482.82	952.83
Break	855.27	900.82	491.01	481.34	606.05	485.61	954.39
Exact	899.23	948.76	530.11	515.42	646.84	523.85	1063.31

Table A.6: G-7 countries, test based on K

AIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	917.23	992.83	598.17	573.30	713.66	559.75	1086.35
Trend	916.49	994.58	607.07	581.52	719.83	564.82	1092.11
Regression	926.50	1000.14	609.56	580.04	723.08	549.58	1094.49
Break	917.71	996.77	608.87	583.50	721.81	570.48	1095.57
Exact	992.54	1062.05	672.41	645.07	785.20	622.17	1142.31
BIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	878.13	953.74	537.38	512.50	654.83	498.96	1050.24
Trend	876.41	954.50	544.74	519.18	659.51	502.49	1055.03
Regression	886.41	960.05	547.23	517.71	662.75	487.25	1057.41
Break	877.13	956.20	545.77	520.39	660.74	507.38	1058.01
Exact	936.63	1006.13	585.45	558.11	701.05	535.21	1090.79

Table A.7: G-7 countries, test based on L

AIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	617.40	640.20	406.57	404.49	434.60	394.74	700.74
Trend	619.71	646.72	414.72	405.36	444.68	403.20	703.32
Regression	622.14	645.84	415.94	407.68	445.88	400.82	702.28
Break	621.34	647.61	416.37	407.46	446.99	407.26	704.24
Exact	687.76	708.05	481.53	474.76	507.00	464.78	764.64
BIC	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Constant	578.31	601.10	345.77	343.69	375.77	333.94	664.63
Trend	579.63	606.64	352.39	343.03	384.36	340.86	666.24
Regression	582.06	605.76	353.60	345.34	385.55	338.48	665.21
Break	580.76	607.03	353.27	344.36	385.92	344.15	666.68
Exact	631.84	652.13	394.57	387.80	422.85	377.82	713.12