

Optimal Design and Quantitative Evaluation of the Minimum Wage *

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Abstract

We study a labor market where firms have private information about their ex-ante heterogeneous productivities, and where search is random. A minimum wage policy based on the Vickrey-Clarke-Groves mechanism delivers full efficiency in the static model. In the dynamic, stochastic model calibrated to the Routine Manual labor market in the United States, a version of the static policy generates sizeable welfare gains. The resulting minimum wage is procyclical, dampening the response of unemployment to aggregate shocks.

Keywords: Minimum Wage Policy, Business-cycle, Vickrey-Clarke-Groves Auction

JEL Codes: J2, J3, J5

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1 Introduction

A common view of minimum wage policy, dating at least back to [Stigler \(1946\)](#), is that it decreases efficiency to the benefit of equity. By contrast, in this paper we examine the role that minimum wage policy can play in promoting efficiency alone. Intuitively, frictional labor markets where firms capture too much of the match surplus generate excessive entry, an inefficiency that can be mitigated by a minimum wage. If firms are ex-ante heterogeneous in productivity, a properly chosen minimum wage will selectively target the lower tail of the distribution and deliver the planner’s allocation. If, additionally, firm productivities are private information and the wage setting protocol is unknown, the policy-maker must simultaneously set the minimum wage and truthfully elicit firms’ maximum willingness to pay. We show that a policy based on the Vickrey-Clarke-Groves mechanism (henceforth, VCG)¹ achieves these goals in a static version of the Diamond-Mortensen-Pissaredes model (henceforth, DMP)² with the aforementioned ingredients. We next extend the model to a dynamic environment with aggregate shocks to evaluate how the optimal minimum wage behaves over the cycle. It is pro-cyclical, rising in booms and decreasing in busts, smoothing the unemployment response to aggregate shocks. Unfortunately, the dynamic version of the policy derived in the static model requires too much information on the part of the policy-maker. However, if we approximate the dynamic policy with its stationary counterpart — the *quasi*-efficient policy — then the information requirement is similar to the one in the static model, and the welfare gains attained in quantitative simulations are still sizeable. In particular, the quasi-efficient policy generates 90% of the welfare gains associated with the optimal minimum wage in the dynamic model with shocks.

Our model of the labor market has three main ingredients. The first ingredient is a frictional labor market with random search, which is standard in the DMP literature. These frictions create a congestion externality as firms’ vacancy filling probabilities fall in the number of total vacancies posted. The second ingredient is ex-ante firm heterogeneity, which creates an asymmetry in the congestion externality. Low productivity firms are equally likely to fill vacancies as their more productive counterparts, but provide a smaller contribution to aggregate output upon matching. The third ingredient is firms’ private information about their productivities, which motivates the policy problem we solve. In the absence of full information, policy-makers must simultaneously elicit private information regarding firms’ productivity while choosing the productivity cut-off below which no firms can operate. It should be noted that the combination of ex-ante firm heterogeneity and private information

¹After [Vickrey \(1961\)](#), [Clarke \(1971\)](#) and [Groves \(1973\)](#).

²After [Diamond \(1982\)](#), [Pissarides \(1985\)](#) and [Mortensen and Pissarides \(1994\)](#).

is key. If productivities are public information, then policy-makers can directly limit entry below the efficient productivity cut-off. Alternatively, if firm heterogeneity is realized ex-post, no minimum wage can deter entry from less productive firms in a targeted manner.

The problem of simultaneously eliciting private information and setting prices (such as the minimum wage) is precisely what the VCG mechanism achieves. In an auction setting, each bidder is charged the cost they impose on their competitors (the VCG price), while ensuring privately-known valuations are truthfully reported. Similarly, in our labor market, the policy we propose charges each firm at least the congestion externality, while ensuring that firms truthfully report their privately-known productivities. Effectively, our policy adjusts the externality by increasing the entry cost of low productivity firms until the social marginal benefit and cost of vacancy creation are equated.

The policy we propose corrects an externality that causes excessive firm entry. In our model, as in all the DMP literature, the existence of this externality is a question of parameter values. Most commonly, the literature assumes that the Hosios condition (after [Hosios, 1990](#)) is satisfied — for convenience and because it typically is not central to the analysis. In our model with ex-ante heterogeneity, the corresponding assumption would be the generalized Hosios condition of [Julien and Mangin \(2017\)](#), which would imply that the unregulated labor market is efficient. In this case, our policy would deliver a minimum wage equal to the market wage of the boundary firm (that is, the minimum wage would simply be non-binding). However, in our quantitative exercises, we find parameter values that imply excessive firm entry. In fact, to our knowledge, within the class of stochastic DMP models only [Hagedorn and Manovskii \(2008\)](#), [Lagos \(2006\)](#) (in an alternative calibration of his model), and our own calibration exercise, do not assume that any form of the Hosios condition holds. All three find parameter values that imply excessive firm entry, even though the models and calibration strategies differ. In the literature review we have an extensive discussion of this issue.

In the static model we build, ex-ante heterogeneous firms make costly vacancy posting decisions, unemployed job seekers are randomly matched across available vacancies, and bargaining determines market wages. The policy we propose elicits firm productivity information from all filled vacancies, computes the unique VCG price, and sets it as the minimum wage. We show that computing the VCG price is equivalent to solving the planner’s problem and deriving the implied optimal minimum wage. Forecasting the outcome of the policy, no firm with productivity below the optimal cut-off chooses to search, and efficiency obtains. It is important to note that the policy is independent of the wage setting protocol. The dynamic model is a straightforward extension of the static model, but with aggregate shocks. In this framework, the optimal minimum wage implied by the planner’s solution (which would be

the VCG price) conditions on the report of the marginal firm and the specific wage setting protocol in the market. The former violates truth-telling, and the latter severely diminishes the generality of the policy. However, the stationary version of the dynamic policy requires neither. This leads us to propose the *quasi*-efficient policy, which is a period-by-period implementation of the stationary policy in the dynamic framework with shocks. We then move to a quantitative evaluation of this policy.

We calibrate our model to hit the wage distribution and a number of average and cyclical moments from the labor market for Routine, Manual jobs in the United States. We find that setting the minimum wage using the quasi-efficient policy increases real income by 11.75%, which is comparable to the 13.11% increase attained with the fully efficient allocation. When setting the minimum wage, the quasi-efficient policy behaves as if the states of the economy are to be constant going forward. However, since the states of the economy fluctuate with the business cycle, the resulting minimum wage is flexible, contrasting with the sluggishness of the minimum wage in the data.

The increase in real income generated by the quasi-efficient policy is driven, in part, by the minimum wage preventing entry of less productive firms. This was the force highlighted in the static version of the model. But part of the increase in real income is the result of having a flexible minimum wage. The minimum wage acts to dampen the entry effects of aggregate shocks, reducing the volatility of unemployment and job finding rates. This dampening does not arise from any insurance motive; agents are risk neutral in the model. However, in the calibration the aggregate component of output is fairly transitory, reducing the planner's incentive to respond to these aggregate fluctuations. By symmetry, the minimum wage increases in recession periods. This pro-cyclicality in the policy is mirrored by pro-cyclical gains. Relative to the benchmark economy, the quasi-efficient policy obtains the highest welfare gains during booms, as it tightens the minimum wage.

Last, we perform two sets of robustness checks on our analysis. First, we simulate the policy operating at monthly, quarterly, and yearly frequencies. In our benchmark case, we solve the model (and hence, the policy) at a weekly frequency to deal with time aggregation issues. However, this implies an unrealistic responsiveness of the minimum wage to the cyclical movements (particularly given our assumption that the aggregate state is known). We find that adjusting the frequency with which the minimum wage is updated has little impact on our results in the benchmark calibration. Relative to the benchmark calibration, the policy yields gains of 11.74%, 11.71% and 11.64%, when operating at monthly, quarterly, and yearly frequencies, respectively.

Second, we re-calibrate the model to the entire U.S. labor market. This stretches the credibility of the assumption of homogeneous workers, but allows for a closer comparison

of our results to those of the quantitative DMP literature on business cycles. The model performs well, reproducing the cyclical targets of [Hagedorn and Manovskii \(2008\)](#) along with other moments from the wage distribution. Imposing the quasi-efficient policy in this economy yields results that are qualitatively similar to the routine manual benchmark but imply welfare gains that are more than one order of magnitude larger. Such large gains are an indirect statement about the implied size of the congestion externality under the calibration, which we take as support for using a more narrowly defined labor market as the benchmark.

Literature review

Policy-makers in our model do not know whether there is excessive firm entry. If there is not, the policy guarantees a non-distortionary minimum wage. The more interesting case however, arises when there is excessive entry, and the policy is able to restore efficiency. The well-known Hosios condition (after [Hosios, 1990](#)) establishes that the DMP model delivers the efficient allocation (that is, there is no excessive entry) when parameters are such that the share captured by workers from bargaining, α , is equal to one minus the elasticity of the matching function, $M(m, n)$, with respect to job postings, n (with m being the number of job seekers). If, instead, the share that workers capture is lower, the labor market has excessively high firm entry. It is common practice in the literature to pick parameters that satisfy the Hosios condition because it is convenient, and calibrating α is typically not crucial (see, for example, [Shimer \(2005\)](#), [Pries and Rogerson \(2005\)](#), and [Pissarides \(2009\)](#) as examples where the Hosios condition is assumed, but the list is far from exhaustive). However, [Hagedorn and Manovskii \(2008\)](#), who develop a model similar to ours, calibrate their model and find that the empirically relevant case is when there is excessive firm entry. They choose the matching function to be $M(m, n) = mn/(m^\kappa + n^\kappa)^{1/\kappa}$, calibrate the bargaining parameter to $\alpha = 0.052$, the matching function parameter to $\kappa = 0.407$, and use market tightness of $\theta = n/m = 0.634$ (the latter combining estimates of [Shimer, 2005](#), and [den Haan et al., 2000](#)). These parameters imply that “one minus the elasticity of the matching function with respect to job postings” is $(1 + \theta^\kappa)^{-1} = 0.546$, a number larger than α . Hence, the empirically relevant case for their model implies excessive entry of firms. Moreover, [Hagedorn and Manovskii \(2008\)](#) also mention that [Hall \(2005\)](#) obtains a $\theta = 0.539$ from direct estimates of the Job Opening and Labor Turnover Survey. Using this number, $(1 + \theta^\kappa)^{-1} = 0.563$, which again implies excessive entry. Another example is [Lagos \(2006\)](#). While in the benchmark calibration the Hosios condition is assumed, an alternative calibration finds $\alpha = 0.1$ when the Hosios condition is not imposed. Since “one minus the elasticity of the matching function with respect to job postings” equals 0.72, the alternative calibration implies excessive entry

as well.³ Finally, in our benchmark calibration, we find that $\alpha = 0.3$ with κ calibrated to 0.3515. Given these numbers, $(1 + \theta^\kappa)^{-1} = 0.619$, which, being larger than the calibrated bargaining power, yields excessive entry. In our later calibration to the U.S. economy, we find $\alpha = 0.05$ while $\kappa = 0.39$, yielding an elasticity of $(1 + \theta^\kappa)^{-1} = 0.544$, and, again, excessive firm entry.

This is not the first paper to think of the minimum wage as an efficiency enhancing policy. Swinnerton (1996), building on the static model of Albrecht and Axell (1984), is the first to formalize the idea that a binding minimum wage can prevent the least productive firms from operating in the market. Related, Flinn (2006) builds a structural DMP model, estimates it in the steady state, and finds that the optimal minimum wage is also positive. Differently than them, we analyze an economy where the government does not know how productive firms are, propose a policy to find the optimal minimum wage, and quantitatively evaluate the gains from pursuing this policy in a dynamic environment with shocks. Ours is also not the first paper to explore the role that the minimum wage plays as a tool to solve information imperfections in the labor market (see Drazen, 1986; Lang, 1987; and van den Berg, 2003). Although the types of informational problems considered in these papers are different than in ours, these works abstract from the question of policy design, and the quantitative evaluation of a positive minimum wage in a dynamic environment with shocks. Galenianos et al. (2011) study the role that the minimum wage has in a directed search model with perfect information and heterogeneous firms, and find that it can exacerbate, not improve, inefficiencies. Their result hinges on the assumption that there is a finite number of firms in the market that have market power, which implies that an inefficiently small number of firms operate. Hence, forcing some firms out makes matters worse. By comparison, in our model there is an excess of firms, and the minimum wage deters entry of less productive ones. Moreover, we introduce a policy to circumvent the lack of information in the market, and find the efficient minimum wage.

Another strand of the literature has explored the role the minimum wage plays as a redistributive policy. Examples of these are Boadway and Cuff (2001), Gorostiaga and Rubio-Ramírez (2007), or, more recently, Lee and Saez (2012). Either explicitly, through government preferences, or implicitly, through curvature in individuals' utility, these papers find that minimum wages can be welfare improving through reduced inequality. In our case, we abstract from these considerations by having utilitarian government preferences and linear utility. Nonetheless, our minimum wage policy also improves welfare through pure gains in net output.

Finally, there is a growing literature that focuses on dynamic mechanisms that yield the

³See Appendix B in Lagos (2006).

fully-efficient allocation — see [Bergemann and Välimäki \(2010\)](#), [Athey and Segal \(2013\)](#) and [Pavan et al. \(2014\)](#) as prominent examples of this literature. By contrast, our paper proposes a period-by-period implementation of the version of a well-known, static mechanism in a dynamic environment. Even though it cannot deliver full-efficiency, we show that this policy induces a quantitatively similar allocation. To the best of our knowledge, no dynamic mechanism that generates full efficiency has been proposed in an environment where, as in our dynamic model, some agents die every period, new agents with privately known valuations enter, and the aggregate state of the economy is stochastic. We believe the approach that we pursue with the quasi-efficient policy — implementing an allocation close to the planner’s solution — is a valuable alternate route.

2 Minimum wage in the United States

In the United States, the Federal minimum wage must be approved by Congress. The slow pace of the political process makes the minimum wage lack flexibility in adjusting to economic fluctuations. In fact, between 1982 and 2017, the Federal minimum wage law has only changed three times, in 1990, 1996, and 2007. The first two changes consisted of two gradual increments each, and the most recent change saw three increments, to \$5.85 (2007), \$6.55 (2008), and \$7.25 (2009), its current value.⁴ State and local legislatures can choose higher minimum wages in their constituencies — for instance, New York and California are phasing in a \$15 minimum wage — but these policies are nonetheless subject to similar sluggishness as at the federal level.

The red line in [Figure 1](#) shows the evolution of the federal nominal minimum wage discussed above. The step-wise nature of the series clearly indicates the infrequent adjustments to the law. The blue line in the figure incorporates variation in state-level minimum wages, and plots the evolution of the population-weighted average nominal minimum wage. The series were computed using data from CPS-MORG and [Vaghul and Zipperer \(2016\)](#).⁵

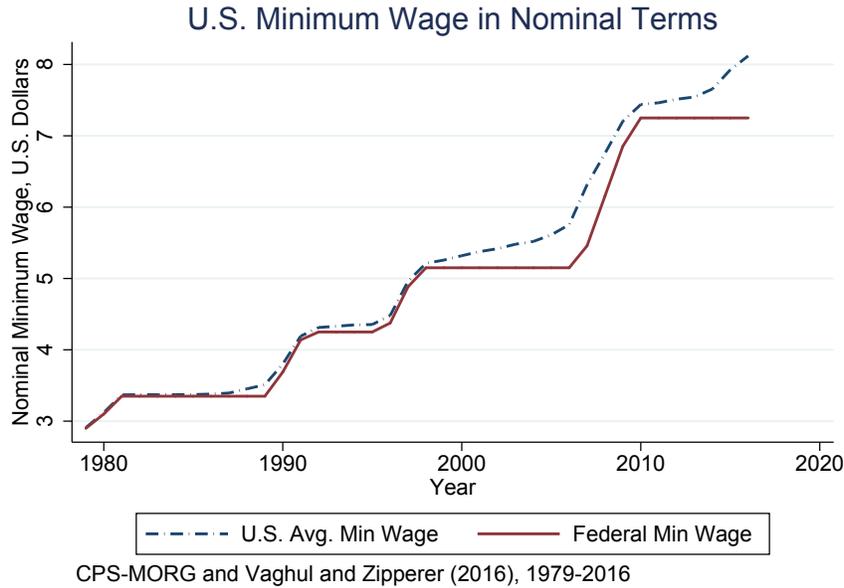
In [Figure 2](#) we plot the evolution of the deflated, population-weighted minimum wage, which is the real counterpart to the blue line of [Figure 1](#). As above, the series exhibits three discrete jumps, coinciding with the changes in the law. The subsequent declines in real value are simply the result of inflation.

In the last two figures we plot the fraction of employed workers receiving the minimum wage for the entire U.S. labor market, [Figure 3](#), and the Routine Manual submarket, [Figure 4](#). These series are constructed for the period 1990-2016, as this is the time period from

⁴See [U.S. Department of Labor \(2011\)](#) §206 of the Fair Labor Standards Act of 1938, as Amended.

⁵See the Online Appendix for details on data construction.

Figure 1:

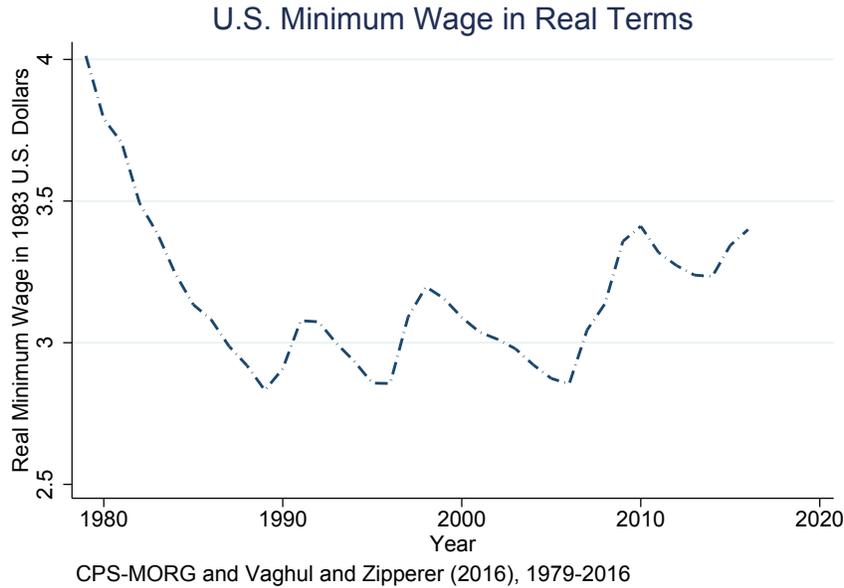


which our target moments will be taken. These graphs are informative of the severity of the constraint implied by minimum wage policy. The picture that emerges in both markets is remarkably similar to Figure 2, implying that the impact of policy changes is larger than that of cyclical fluctuations on the fraction of minimum wage earners. We attribute the few discrepancies between the two figures — early 1990’s and early 2010’s — to recession periods.

3 The VCG mechanism in the labor market

In an auction setting, the goal of the Vickrey-Clarke-Groves (VCG) mechanism is to achieve efficiency through price setting when bidder valuations are private information. The intuition is to compare the value generated when a bidder participates in an auction against when she does not — her participation externality. If the bidder’s private valuation is larger than the externality she causes, allocating her the good is socially optimal. To keep the analogy, in our labor market, firms are bidders, and workers are goods. If the value generated by the match is larger than the participation externality of the firm, her participation in the market is socially optimal. Absent frictions, workers would be matched with high productivity, high paying firms, and the outcome would be efficient. However, in a labor market with random search, low productivity firms are equally likely to fill a vacancy as their more productive counterparts, but provide a smaller contribution to aggregate output upon matching. Setting

Figure 2:



the minimum wage using a version of the VCG mechanism (our proposed policy) allows only those firms whose participation generates a sufficiently high value to enter the market.

We build the intuition of our policy with a parsimonious example. We start with three heterogeneous firms, two homogeneous workers and no search frictions. Then, we show that adding search frictions into the model may generate an inefficient allocation, but that setting the minimum wage using our policy restores efficiency. Next, we add non-market value for workers, search costs, many firms, many workers, and show that the result from the simple example remains. Lastly, we formally define the policy and show that it delivers the efficient allocation in our static model.

3.1 Three heterogeneous firms, two workers and no frictions

We start with a frictionless labor market. There are three firms with productivities $p_1 > p_2 > p_3$ and two homogeneous workers. In equilibrium, firms 1 and 2 hire a worker, firm 3 is left vacant, and the equilibrium is efficient. To implement our policy in this labor market, we compute the VCG price for each firm and select the maximum of these values as the minimum wage.

Recall that the VCG price is computed as the externality caused by a firm participating in the market. Start with firm 1. If she does not participate, firms 2 and 3 each hire one worker, and the resulting output from *other* firms is $p_2 + p_3$. If, instead, firm 1 participates, the output generated by other firms is p_2 , as firm 1 hires away firm 3's worker. We can

Figure 3:



compute the externality of firm 1’s participation as $W_1 = (p_2 + p_3) - p_2 = p_3$. Similarly, firm 2’s externality is $W_2 = p_3$. For firm 3 the story is different. If she does not participate, the output generated by other firms is $p_1 + p_2$; if she does participate, the output generated is still $p_1 + p_2$, as she hires no worker with an equilibrium wage $p_1 > p_2 \geq w > p_3$. Hence, firm 3’s externality is $W_3 = (p_1 + p_2) - (p_1 + p_2) = 0$.

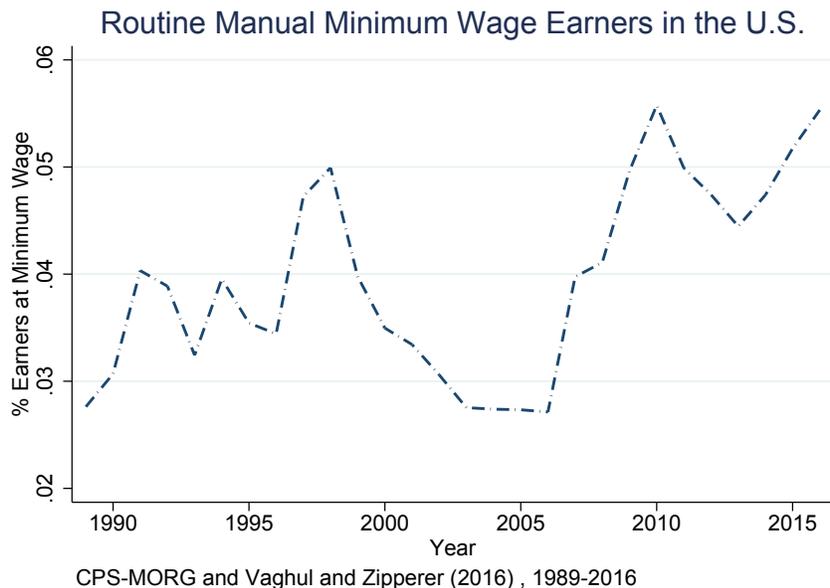
Let $\hat{w} = \max\{W_i\}_{i=1,2,3} = p_3$ be the maximum participation externality generated by a firm. If we make each firm pay \hat{w} when participating in the market, then only firms 1 and 2 are strictly willing to participate. Firm 3 is indifferent, as is the social planner, who values her participation at exactly 0. Hence, a policy that makes firms pay \hat{w} is not necessary, because in its absence, the equilibrium is already efficient. As we shall see next, though, adding search frictions creates inefficiencies that can be restored by this policy.

Last, note that when determining each W_i , we do not use the report of firm i itself (as p_i does not show up in the calculation). This feature is key to ensure that firms truthfully report their privately observed p_i .

3.2 Adding search frictions: rationale for a minimum wage

Suppose we introduce a friction in the previous economy: matches occur through matching technology $M(m, n)$, where m is the number of workers, and n is the number of firms. Hence, the vacancy filling rate is $M(m, n)/n$. The matching function satisfies standard assumptions

Figure 4:



of homogeneity of degree 1, and curvature.⁶

Following the logic from before, when firm 1 does not participate, the *expected* output created by other firms is the vacancy filling rate times the output generated by participating firms. The vacancy filling rate is $M(2, 2)/2$ because there are two workers and two firms in the market. The output generated by other participating firms is $p_2 + p_3$ because all firms but firm 1 participate. Hence, total expected output is $M(2, 2)/2 \times (p_2 + p_3)$. Likewise, when firm 1 participates, the expected output created by other firms is $M(2, 3)/3 \times (p_2 + p_3)$. Note that the impact of firm 1 participating is seen through the vacancy filling rate. Hence, firm 1's participation affects all competing firms equally through the congestion externality. For comparison, recall that absent frictions, firm 1's participation had no impact on firm 2, but left firm 3 vacant. With these expressions, we can compute the externality of firm 1, which is equal to

$$W_1 = M(2, 2) \times \frac{p_2 + p_3}{2} - M(2, 3) \times \frac{p_2 + p_3}{3}. \quad (1)$$

We now focus our attention on firm 3. If firm 3 does not participate, the output generated by other firms is $M(2, 2)/2 \times (p_1 + p_2)$; if she participates, the output generated by other firms is $M(2, 3)/3 \times (p_1 + p_2)$. Hence, firm 3's participation externality is $W_3 = (M(2, 2)/2 - M(2, 3)/3) \times (p_1 + p_2)$.

Firm 3 will participate if it is profitable for her to do so. Let w_3 refer to the wage she

⁶In particular, we assume that $\frac{\partial M(m, n)}{\partial n} < 0$.

pays to the worker. Then, expected profits from participating are

$$\pi(p_3) = \frac{M(2, 3)}{3}(p_3 - w_3), \quad (2)$$

which will be positive as long as $p_3 \geq w_3$.

However, from an efficiency standpoint, firm 3 should only participate if the social value generated by her participation, Z_3 , is large enough. That is, if

$$Z_3 = M(2, 3) \times \frac{p_1 + p_2 + p_3}{3} - M(2, 2) \times \frac{p_1 + p_2}{2} > 0. \quad (3)$$

Note that even though firm 3 has the smallest productivity, under search frictions her participation may still be desirable. This occurs when the increased number of matches from her participation outweighs the decline in average productivity. Namely, Z_3 may be positive.⁷ However, even if firm 3's participation in the market is inefficient, she may still decide to participate because her expected profits are positive — equation (2) may be positive even if equation (3) is negative.

One way to guarantee that firm 3 participates only if it is socially desirable, is to impose a wage that aligns firm 3 profits with social gains. Setting w_3 to be

$$w_3 = \frac{3}{M(2, 3)} \times \left(\frac{M(2, 2)}{2} - \frac{M(2, 3)}{3} \right) \times (p_1 + p_2) \equiv w_3^{VCG}, \quad (4)$$

makes equations (2) and (3) coincide, where $E(\pi(p_3)) = Z_3$. Moreover, note that the expression in equation (4) exactly coincides with firm 3's externality in this environment, adjusted by the inverse of the vacancy filling rate.

We define this adjusted externality as the VCG price. Prices for firms 1 and 2 can be similarly derived, with the only difference being the sum of other firms' productivities at the right part of the expression. Since the sum of other firms' productivities is highest for the lowest productivity firm, setting the minimum wage equal to the maximum of the VCG prices yields $\hat{w} = w_3^{VCG}$. As before, the VCG price for firm i , w_i^{VCG} , does not use information on p_i , so truth-telling obtains.

3.3 Efficiency with the minimum wage equal to the VCG price

We now show that setting the minimum wage using our policy delivers efficiency in a more general framework. To this end, we incorporate a non-market value for workers, b , and vacancy posting costs for firms, s . Moreover, the economy now features many homogeneous

⁷Note that even though $(p_1 + p_2 + p_3)/3 < (p_1 + p_2)/2$, we have that $M(2, 3) > M(2, 2)$.

workers, m , and many heterogeneous firms, n , with productivities $p_1 > p_2 > \dots > p_n > b$.

In this framework, the VCG price for a given firm i — the generalized version of equation (4) — is equal to

$$w_i^{VCG} = \frac{n}{M(m, n)} \left(M(m, n-1) \left(\frac{1}{n-1} \sum_{\forall j/i} p_j - b \right) - M(m, n) \left(\frac{1}{n} \sum_{\forall j/i} p_j - b \right) \right). \quad (5)$$

In Appendix A.1 we show how to derive equation (5). The policy then selects the minimum wage to be

$$\hat{w} = \max\{w_i^{VCG}\}. \quad (6)$$

We now derive our primary result in this environment: when setting the minimum wage according to equation (6), efficiency obtains. To do so, we first characterize the planner's solution through an optimal cut-off productivity. We then show that the policy obtains the identical cut-off.

Since redistribution is not valued in our economy, the social planner only seeks to maximize net output. As output increases in productivity, the planner selects the x^* -highest firms to participate in the market, where firm $x^* < n$ has the optimal cut-off productivity. To find x^* , consider

$$\Omega_x = M(m, x) \left(\frac{1}{x} \sum_{j=1}^{j=x} p_j - b \right) - xs, \quad (7)$$

where the first term is the number of matches multiplied by the average output produced by the x -largest firms, and the second term is the cost incurred by having x firms searching. We assume that the matching function and firm productivities are such that $\Omega_{x+1} - \Omega_x$ is initially positive, monotonically decreasing, and crosses zero. Then, x^* is characterized by $\Omega_{x^*} - \Omega_{x^*-1} \geq 0$, and $\Omega_{x^*+1} - \Omega_{x^*} < 0$.

In the market economy, firm i 's expected profits are equal to the vacancy filling rate times operating profits (output minus wage), net of search costs. Namely,

$$\pi(p_i) = \frac{M(m, x^M)}{x^M} (p_i - w_i) - s, \quad (8)$$

where x^M is the number of firms operating in the market, and w_i is the salary that firm i pays to the worker. When the minimum wage is determined by equation (6), the marginal

firm in this economy makes profits that satisfy

$$\begin{aligned}
\pi(p_M) \geq 0 &\Leftrightarrow \frac{M(m, x^M)}{x^M} (p_M - \hat{w}) \geq s \Leftrightarrow \\
&- M(m, x^M - 1) \left(\frac{1}{x^M - 1} \sum_{\forall j/M} p_j - b \right) \\
&+ M(m, x^M) \left(\frac{1}{x^M} \left(\sum_{\forall j/M} p_j + p_M \right) - b \right) \geq s \\
&\Leftrightarrow \Omega_M - \Omega_{M-1} \geq 0,
\end{aligned}$$

and, using similar algebra, we get that $\Omega_{M+1} - \Omega_M < 0$.

Note that the efficient marginal firm, x^* , and the marginal firm that the policy selects, x^M , are characterized by $\Omega_{x^i} - \Omega_{x^{i-1}} \geq 0$ and $\Omega_{x^{i+1}} - \Omega_{x^i} < 0$, for $i = \{*, M\}$. Since $\Omega_{x^i} - \Omega_{x^i}$ is monotonically decreasing, it must be true that $x^M = x^*$. Hence, the policy implements the efficient allocation in this richer environment.

3.4 Continuum of firms and workers, and the policy

We now extend the model to the continuum of agents case, which will prove a convenient step toward doing the quantitative exercise in the following Section. Here, we formally define our policy and extend the efficiency results.

It is useful to first derive the VCG price in this framework — that is, the counterpart to equation (6) for a continuum of workers and firms. We do so in the next Lemma.

Lemma 1 *The VCG price with a mass n of firms and m of workers is given by*

$$w^{VCG} = \int p(x) dJ(x) - M_2(m, n) \frac{n}{M(m, n)} \left(\int p(x) dJ(x) - b \right). \quad (9)$$

Proof. See Appendix A.2. ■

Having a continuum of atomistic agents implies an equal VCG price for all firms — hence, the maximum VCG price is also given by equation (9). Moreover, it delivers the efficient allocation, a result that we formalize in the following Lemma.

Lemma 2 *The social planner allocation can be implemented with a minimum wage equal to*

$$\hat{w} = w^{VCG}. \quad (10)$$

Proof. See Appendix A.2. ■

The results in Lemmas 1 and 2 are very useful. They imply that if the policy-maker sets the minimum wage equal to the VCG price, then the efficient allocation is obtained. However, to compute this price, the planner must be able to truthfully elicit information on firm productivities. We now show that it can.

To discuss implementing the policy, we must first introduce a notion of intra-period timing. Within our single period, all firms and workers who choose to may search. Then, the policy-maker sets the minimum wage, and production and payment follow. Note that the government can respond to search decisions, implying that agents must form expectations on what government policy will be.

We assume there is a continuum $m < 1$ of workers, and $N = 1$ of firms, with privately-known productivities $p \in [\underline{p}, \bar{p}]$, distributed according to $J(p)$. As before, $\underline{p} > b$. Define $\delta(p)$ as the binary participation decision that a firm with productivity p makes, and $\gamma(p)$ the report she sends to the policy-maker upon matching. Then, $S(p) = (\delta(p), \gamma(p)) \in \mathcal{S}$ denotes a strategy profile for this firm, and $n = \int \delta(x) dJ(x) \leq N$ is the total mass of searching firms.

The policy, $\Gamma = (S, t)$, maps the space of strategy profiles S to a payment function $t : R_+ \times \mathcal{S} \rightarrow R_+$ denoting the minimum wage to be paid by each firm. For any two identical reports γ , this lower bound is exactly the same. In fact, as shown above, the minimum wage imposed by the policy will be identical for all reports. Formally,

Definition 3 *The policy Γ is a strategy profile S and payment function, $t : R_+ \times \mathcal{S} \rightarrow R_+$, such that, for all γ ,*

$$t(\gamma, S) = \int \delta(x) \gamma(x) dJ(x) - M_2(m, n) \frac{n}{M(m, n)} \left(\int \delta(x) \gamma(x) dJ(x) - b \right).$$

It is important to remember that this payment function is a minimum, not the actual payment to workers. In fact, the salary that workers receive, and that firms pay, is equal to $w^t(p) = \max\{t(\gamma(p), S), w(p)\}$, where $w(p)$ is the market wage. In an effort to map the VCG mechanism to our policy, we are being loose with the notion of a payment function. While our minimum wage indirectly maps reports to allocations, the policy relies on market forces to achieve these outcomes.

The policy works by making firms' cost of participating in the labor market at least as large as the social cost of their participation — which is a re-interpretation of the VCG intuition. Moreover, the policy generates truthful reports by firms because outcomes depend on averages rather than individual reports. Let

$$\pi(p, S) = \delta(p) \left(\frac{M(m, n)}{n} (p - w^t(p)) - s \right)$$

be the profits that a firm with productivity p attains by choosing the truth-telling strategy profile S . Then, consider any other strategy profile $\tilde{S}_p = (\hat{\delta}(p), \hat{\gamma}(p))$. An equilibrium induced by policy Γ has to satisfy that the profits from firm p will be at least as large by choosing the truth-telling strategy as by choosing any other. Formally, $\pi(p, S) \geq \pi(p, \tilde{S}_p)$ for all $\tilde{S}_p, p \in \mathcal{P}$.

In the following Theorem, we establish that the policy we propose induces the truth-telling equilibrium, and it coincides with the social planner solution.

Theorem 4 *Policy Γ truthfully implements the planners solution.*

Proof. See Appendix [A.3](#). ■

In this Section we have shown how the policy induces efficiency in a static labor market with many elements common to search models. Next, we investigate whether these results are preserved in a dynamic environment with shocks.

4 Dynamic model

In this Section we extend the model to a dynamic, infinite horizon environment, and show how to embed the policy in it. Since the goal is to perform a quantitative analysis, the dynamic version of the model in this Section is written in recursive form. The timing of the model is as follows: upon each period beginning i) the aggregate productivity shock z is realized; ii) existing matches produce; iii) a fraction δ of existing matches are destroyed, moving workers into unemployment; iv) u workers and n firms search; v) $M(u, n)$ new matches are randomly formed; and vi) the government sets the minimum wage.

4.1 Decentralized economy

The government in this economy follows a minimum wage policy \hat{w} , which can be positive or zero, and may depend on any components of its information set. We assume that the matching technology M , the unemployment level u , the non-market value of unemployment b , and the vacancy posting cost s belong to this information set, while the firm productivity distribution $J(p)$, the wage setting protocol between workers and firms, $w(p)$, and the cut-off firm productivity, p^* do not. In what follows, and for clarity of exposition, we will write \hat{w} ignoring its arguments. However, it will be a function. Also for convenience, we do not make the dependence of value functions on minimum wage policy explicit.

There is a mass 1 of workers in the economy who are either employed or unemployed. We do not allow for on-the-job search, and assume all workers must wait one period to search after a job destruction episode. Time is discounted at rate β .

The value of being unemployed, U , is the same for all workers, depends on aggregate states z and u , and is given by

$$U(z, u) = b + \beta E \left[\lambda(z', u') \int_{p^*(z', u')} W(x, z', u') dJ(x) + (1 - \lambda(z', u')) U(z', u') \right], \quad (11)$$

where λ is the endogenous contact rate and $W(x)$ is the employment value of matching with a firm with productivity x . The notation $'$ denotes next period variables.

The value of employment at firm p , $W(p)$, is given by

$$W(p, z, u) = \max\{\hat{w}, w(p, z, u)\} + \beta E [(1 - \delta)W(p, z', u') + \delta U(z', u')]. \quad (12)$$

As in the static model, there is a mass $N = 1$ of firms, with privately-known productivities $p \in [\underline{p}, \bar{p}]$, and $\underline{p} > b$. Firms discount the future at the same rate β as workers. The value of an operating firm with productivity p is

$$\Pi(p, z, u) = z + p - w(p, z, u) + \beta(1 - \delta)E [\Pi(p, z', u')]. \quad (13)$$

The free entry condition pinning down the marginal productivity p^* is

$$\frac{M(u, \bar{J}(p^*(u, z)))}{\bar{J}(p^*(u, z))} \beta \Pi(p^*(u, z), u, z) = s, \quad (14)$$

where $\bar{J} = 1 - J$.

We are now ready to define an equilibrium for this economy.

Definition 5 *Given a minimum wage function, \hat{w} , an aggregate productivity process, z , and the unemployment level, u , a **recursive equilibrium** consists of a cut-off firm productivity, $p^*(z, u)$, value functions $U(z, u)$, $W(p, z, u)$, $\Pi(p, z, u)$, and market wage function $w(p, z, u)$, such that equations (11) – (14) are satisfied, and next period unemployment is given by $u' = u - M(u, \bar{J}(p^*(u, z))) + \delta(1 - u)$.*

Given an arbitrary minimum wage policy \hat{w} , we can conveniently combine firm's profits, equation (13), with the free entry condition, equation (14), to express the cut-off productivity implicitly as

$$\frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)} \beta \left(\frac{p^*}{1 - \beta(1 - \delta)} + E \left[\sum_{s=0}^{\infty} (\beta(1 - \delta))^s (z^{(s)} - \max\{w^{(s)}, \hat{w}\}) \right] \right) = s. \quad (15)$$

In the next Section we derive the optimal cut-off productivity, and compare the two expressions.

4.2 Social Planner problem

Given that agents have linear utility, the social planner chooses the cut-off productivity firm, p_{SP}^* , to maximize the present value of net output. A given period's net output is the sum of output produced by operating firms plus the flow from non-market activities of unemployed workers less the cost of all vacancies posted. Hence, the social planner's problem is given by

$$\begin{aligned} \Omega(\hat{p}, z, u) &= \max_{p_{SP}^*} \left\{ (z + \hat{p})(1 - u) - s\bar{J}(p_{SP}^*) + ub + \beta E[\Omega(\hat{p}', z', u')] \right\} \\ \text{subject to: } u' &= u - M(u, \bar{J}(p_{SP}^*)) + \delta(1 - u), \\ \hat{p}' &= \frac{\hat{p}(1 - \delta)(1 - u) + M(u, \bar{J}(p_{SP}^*))E(p|p > p_{SP}^*)}{(1 - \delta)(1 - u) + M(u, \bar{J}(p_{SP}^*))}, \\ \log(z') &= \mu \log(z) + \epsilon, \text{ where } \epsilon \sim N(0, \sigma), \end{aligned} \tag{16}$$

where $\bar{J}(p_{SP}^*) = \int_{p_{SP}^*}^{\hat{p}} dJ(x)$ and $E(p|p > p_{SP}^*) = \int_{p_{SP}^*}^{\hat{p}} xdJ(x) / \int_{p_{SP}^*}^{\hat{p}} dJ(x)$. The solution to problem (16) implicitly characterizes the optimal cut-off productivity policy, $p_{SP}^*(\hat{p}, z, u)$, which we characterize in the following Lemma.

Lemma 6 *The optimal cut-off productivity, $p_{SP}^*(\hat{p}, z, u)$, is given by the implicit equation*

$$\begin{aligned} & \beta M_2(u, \bar{J}(p_{SP}^*)) E \left[\begin{aligned} & z' + \hat{p}' - b + \beta \frac{\Delta_p^{(2)}(1 - \delta) + M_1(u, \bar{J}(p_{SP}^*))\Phi_p^{(2)}}{1 - \tilde{\beta}_M} \\ & + \sum_{i=2}^{\infty} \prod_{j=1}^{i-1} \tilde{\beta}_{SP}^{(j)} \left(\begin{aligned} & z^{(i)} + \hat{p}^{(i)} - b \\ & + \beta \frac{\Delta_p^{(i+1)}(1 - \delta) + M_1(u^{(i)}, \bar{J}(p_{SP}^*)^{(i)})\Phi_p^{(i+1)}}{1 - \tilde{\beta}_M} \end{aligned} \right) \end{aligned} \right] \\ &= \frac{\beta}{1 - \tilde{\beta}_M} \left(M_2(u, \bar{J}(p_{SP}^*))\Phi_p^{(0)} + \frac{M(u, \bar{J}(p_{SP}^*))}{\bar{J}(p_{SP}^*)} (E(p|p > p_{SP}^*) - p_{SP}^*) \right) + s \end{aligned} \tag{17}$$

where $\tilde{\beta}_M = \beta(1 - \delta)$, $\tilde{\beta}_{SP}^{(j)} = \beta(1 - \delta - M_1(u^{(j)}, \bar{J}((p_{SP}^*)^{(j)})))$, $\Delta_p^{(i+2)} = \hat{p}^{(i+1)} - \hat{p}^{(i+2)}$, $\Phi_p^{(i+2)} = \hat{p}^{(i+2)} - E(p^{(i+1)})$.

Proof. See Appendix A.4. ■

The term β_M in equation (17) refers to the effective discount factor by the market, that is, the actual discount factor, β , times the probability of survival, $(1 - \delta)$. Similarly, the term β_{SP} refers to the effective discount factor of the social planner, which modifies the market discount factor by incorporating the marginal impact on match creation of a change in the unemployment pool, summarized by the term $M_1(u, \bar{J}(p))$. Terms Δ_p and Φ_p refer to changes in average firm productivity and the difference between next period's aggregate productivity and average productivity of new matches, respectively.

Equation (17) characterizes the optimal cut-off productivity in the dynamic model, which captures the impact of a marginal firm participating. The left hand side contains the discounted gains from the marginal firm participating, plus the compositional effects on productivity due to the evolution in unemployment. The right hand side contains the discounted loss from the marginal firm producing (instead of the average), the vacancy posting cost, plus the compositional effects on productivity due to the change in vacancies.

The market equilibrium from Definition 5 does not deliver efficient entry: p^* from equation (15) is different than p_{SP}^* from equation (17), unless the minimum wage takes special values. Next, we show that there is an optimal minimum wage, which guarantees that the two cut-offs are the same.

4.3 Efficiency in the market

In the economy we just described, efficiency is ensured as long as $p^* = p_{SP}^*$. For a minimum wage to implement this, it must align equation (15) with equation (17) in all states. In the following Theorem we characterize this optimal minimum wage.

Theorem 7 *A minimum wage equal to*

$$\hat{w} = \frac{1}{1 - \tilde{\beta}_M} E(p|p > p^*) + \left(z + E \left[\sum_{s=1}^{\infty} \tilde{\beta}_M^s (z^{(s)} - w^{(s)}) \right] \right) + \frac{\bar{J}(p^*)}{M(u, \bar{J}(p^*))} \times E \left[\begin{aligned} & \frac{1}{1 - \tilde{\beta}_M} (M_2(u, \bar{J}(p^*)) \Phi_p^{(0)}) \\ & - M_2(u, \bar{J}(p^*)) \left(z' + \hat{p}' - b + \beta \frac{\Delta_p^{(2)}(1 - \delta) + M_1(u, \bar{J}(p^*))' \Phi_p^{(2)}}{1 - \tilde{\beta}_M} \right) \\ & - M_2(u, \bar{J}(p^*)) \sum_{i=2}^{\infty} \prod_{j=1}^{i-1} \tilde{\beta}_{SP}^{(j)} \left(\begin{aligned} & z^{(i)} + \hat{p}^{(i)} - b \\ & + \beta \frac{\Delta_p^{(i+1)}(1 - \delta) + M_1(u^{(i)}, \bar{J}(p^*)^{(i)}) \Phi_p^{(i+1)}}{1 - \tilde{\beta}_M} \end{aligned} \right) \end{aligned} \right] \quad (18)$$

where $\tilde{\beta}_{SP}^{(j)} = \beta(1 - \delta - M_1(u^{(j)}, \bar{J}((p^*)^{(j)})))$, $\tilde{\beta}_M = \beta(1 - \delta)$, $\Delta_p^{(i+2)} = \hat{p}^{(i+2)} - \hat{p}^{(i+1)}$, $\Phi_p^{(i+2)} = \hat{p}^{(i+2)} - E(\hat{p}^{(i+1)})$, ensures that the market allocation is efficient.

Proof. Combine the expression for the equilibrium cut-off firm in the market, equation (15), with the expression for optimal entry of the social planner, equation (17). Using $p^* = p_{SP}^*$, and rearranging, we obtain equation (18). ■

To implement the planner's allocation, the policy must set the minimum wage according to equation (18). However, this requires information on the wage setting protocol — see the first line of that equation — which we have assumed the policy-maker does not have.

Furthermore, even if we were to relax that restriction, the minimum wage would be conditioned directly on the report of the marginal firm, violating incentive compatibility. Hence, the policy cannot implement the planner's allocation.

As we shall see next, these problems disappear when we look at the steady state version of equation (18).

4.4 The Quasi-Efficient Policy

In a stochastic world, a firm paying the minimum wage today may pay the market wage tomorrow. The policy-maker must then know the wage setting protocol to compute profits for the states in which the minimum wage is not binding. However, in the steady state, a firm paying the minimum wage today will always pay the minimum wage going forward. As a consequence, the policy is incentive-compatible once again. Collapsing the stochastic components of equation (18) we obtain the optimal minimum wage in the steady state.

Corollary 8 *In the steady state, a minimum wage characterized by*

$$\hat{w} = \int p(x)dJ(x) - \frac{1 - \tilde{\beta}_M}{1 - \tilde{\beta}_{SP}} M_2(m, n) \frac{n}{M(m, n)} \left[\int p(x)dJ(x) - b \right] \quad (19)$$

where $\int p(x)dJ(x) = z + E(p|p > p^*)$, $n = \bar{J}(p^*)$, $m = u$, $\tilde{\beta}_{SP} = \beta(1 - \delta - M_1(u, \bar{J}(p^*)))$, $\tilde{\beta}_M = \beta(1 - \delta)$, ensures that the market allocation is efficient.

Proof. It is a direct consequence of equation (18), with $u = u'$, $z = z'$, $\hat{p} = \hat{p}'$, $\Phi_p^{(i)} = 0$, $\Delta_p^{(i)} = 0$. ■

Note that equation (17) is almost identical to the expression for the optimal minimum wage in the static model, equation (9). The only difference is that the former has a ratio in the second term that is not present in the latter. This ratio adjusts for the marginal impact of entry on future unemployment, highlighting the dynamic concerns that the social planner takes into account.

A policy that used equation (17) in a stochastic environment would be both dynamic and incentive-compatible. It would fluctuate with aggregate conditions in a myopic sense, as if the policy-maker believed that the aggregate state in each period were to stay constant going forward. Since information constraints imply that the policy-maker cannot implement the fully efficient allocation, we propose this myopic policy to set the minimum wage. We refer to it as the quasi-efficient policy.

The question of how similar the allocation derived by the quasi-efficient policy is to the planner's solution is quantitative in nature. We explore the answer to it in the following Sections.

5 Calibration

Random (as opposed to directed) search is a stark assumption in the DMP literature, and is necessary for the congestion externality we study to arise. Outside of any model, it seems plausible that search is neither fully directed nor fully random, but rather features some mixture of both. We proxy this idea by using a narrowly defined labor market as our benchmark. In particular, our benchmark calibration focuses on a set of occupations classified as Routine Manual by [Cortes et al. \(2016\)](#), building on recent work in the labor literature that groups occupations by similarity of skill requirements. We interpret this sub-market as if workers may direct their search to Routine Manual jobs, while preserving completely random search within this set of occupations.

The stochastic model has twelve parameters that must be chosen. Our calibration approach will set two exogenously and internally calibrate the other ten in a two-step process. Solving the model at a weekly frequency we set $\beta = .99^{1/12}$ as in [Hagedorn and Manovskii \(2008\)](#). We set the exogenous separation rate to be $\delta = 0.006$, from [Cortes et al. \(2016\)](#).

The next set of six parameters determine steady state (or average) values of the model. Each is presented with an associated target although all targets are matched jointly. The posting cost s targets a proportional cost of 0.583 of weekly output for the average firm, following [Hagedorn and Manovskii \(2008\)](#). We assume firm level productivities are log-normally distributed according to (μ_p, σ_p) , and these two parameters, along with the flow value of utility from unemployment b , jointly target the tenth, fiftieth and ninetieth average wage deciles for Routine Manual workers in the CPS-MORG between 1990 and 2016. Deflated at 1980 prices, these yield targets $w_{10} = 3.76$, $w_{50} = 6.82$, and $w_{90} = 13.15$.⁸ Average aggregate productivity \bar{z} is set to target a finding rate $f = 0.064$ in the Routine Manual labor market, a value also taken from [Cortes et al. \(2016\)](#). The sixth parameter, the matching function parameter κ , is used to target average labor market tightness θ (the ratio of vacancies to unemployment). We construct a measure of θ for the Routine Manual sub-market directly from the Job Openings and Labor Turnover Survey (JOLTS), using industry level proportions of Routine Manual jobs computed from the CPS-MORG. The resulting estimate yields $\theta = 0.276$. For comparison, if we calibrate the model to the whole U.S. labor market using $\theta = 0.634$ from [Hagedorn and Manovskii \(2008\)](#), we obtain a value of $\kappa = 0.39$. Holding κ (and hence the matching technology) constant, along with $f = 0.64$, the implied labor market tightness for Routine Manual jobs would be $\theta = 0.186$, where $\theta = (f^\kappa / (1 - f^\kappa))^{(1/\kappa)}$.

⁸It is worth noting that we load all heterogeneity in wages onto what we term firm productivity. In [Appendix A.5](#) we show that our model is isomorphic to one with both worker and firm heterogeneity; all steady state and business cycle dynamics will be driven by the expected distribution of match productivities, which in this case is equivalent to the distribution (μ_p, σ_p) that we calibrate.

The similarity of these values makes us comfortable that our computed θ is reasonable.

The remaining four parameters are matched to simulated output from the stochastic model. The minimum wage \underline{w} , rather than directly targeted from the data, is set to match the average fraction of employed workers for whom the minimum wage is binding. For the Routine Manual labor market during the period 1990-2016, this number is 3.95%. We assume that wages are set according to Nash bargaining, and the bargaining power of workers, α , is used to target the elasticity of wages with respect to productivity in the data, $w_{ep} = 0.778$. For the Routine Manual labor market, the construction of these values is described in the Online Appendix. The aggregate shock z is assumed to be AR(1) and is discretized on a ten point grid. The persistence, ρ_z , and standard deviation, σ_z , of this process are set to match the autocorrelation and standard deviation of measured aggregate productivity, $\rho_D = 0.701, \sigma_D = 0.106$ (see Online Appendix). Recall that aggregate productivity in the model is $(z + \hat{p})/e$, so there is not a direct mapping from, for example, ρ_z to ρ_D . For all cyclical moments we sample the simulated data at a quarterly frequency to be consistent with their empirical counterparts.

Taking β and δ as given, the calibration proceeds in two steps. For a given guess of α , \underline{w} , ρ_z and σ_z , we globally search over the remaining parameters to minimize a loss function that equally weights the percentage deviation of each steady state moment from its target. We then take all ten parameters (four guesses and six conditionally-matched) to simulate the stochastic model over 300,000 weeks (throwing out the first 30,000) and compute the remaining four moments. Repeating this procedure with consecutive guesses, we search globally over the first four parameters to minimize a similar loss function in four dimensions. See the Online Appendix for a more detailed description of the calibration algorithm. A summary of parameters, moments and targets for our benchmark calibration to the Routine Manual labor market is provided below in Table 1.

5.1 Discussion

One result of our calibration approach is to generate separately a distribution of idiosyncratic firm productivities and an aggregate shock process. Our values imply a relatively large impact of the aggregate shock on average firm output. A “peak to trough” comparison of the upper and lower supports of the discretized z distribution implies that output of the average firm varies by 50.4%. It is worth remembering of course that this is also the maximum variation possible and that these extremes of the z -distribution are not often reached.

We can also see the impact of firm heterogeneity over cycles through the calibration of the aggregate process itself. The weekly autocorrelation of $\rho = 0.9165$ implies a quarterly persis-

Table 1: Parameters and calibration targets, benchmark

	Meaning	Value	Target	Model	Data
s	Posting cost	10	$s/(\hat{p} + \bar{z})$	0.539	0.583
κ	Match fun	0.3515	v/u	0.25	0.276
\underline{w}	Min wage	1.6	Frac of earners at \underline{w}	0.0369	0.0395
σ_p	$\text{var}(\log(p))$	0.5619	w_{90}	13.45	13.15
μ	mean $\log(p)$	2.9939	w_{50}	5.53	6.82
b	Non-market	-43.2387	w_{10}	2.88	3.76
\bar{z}	$E[z]$	-58.5336	Finding Rate	0.064	0.064
α	Barg. power	0.3	Elast. wages to p	0.772	0.778
ρ	Autocorr z	0.9165	Autocorr Y/L (Q)	0.7115	0.7071
σ_z	SD z	0.0085	SD Y/L (Q)	0.1082	0.1055

tence of $0.9615^{12} = 0.3512$, well below the simulated quarterly persistence of average output 0.7155. This sluggish response of measured productivity (output per worker), and indeed the economy itself, to aggregate shocks, is a product of ex-ante heterogeneity. Heterogeneity of any form implies that even when, in a boom, less productive firms are entering, they have a fairly small initial effect on output per worker. It takes time for the composition effects to be fully seen. The ex-ante heterogeneity of firms however gives an additional slowing force. With no heterogeneity or only ex-post heterogeneity, vacancy posting depends solely on market tightness, which governs the matching probability. This is because, conditional on matching, each vacancy has the same expected profits. With ex-ante heterogeneity this is no longer true: when unemployment is high, tightening the labor market requires consecutively worse firms to enter, and this slows down the responsiveness of vacancy posting to changes in z .

6 Results

We run two exercises and compare them to the results from the benchmark calibration, where the minimum wage is set to be the one currently in use. The first experiment implements the quasi-efficient policy described in Section 4.4. The second experiment computes the fully efficient allocation as a guideline for potential first-best gains, as described in Section 4.3. Summary values from the two experiments and the benchmark are compared in Table 2.

We find that the quasi-efficient policy generates an increase of 11.75% in average welfare with respect to the benchmark economy. Of greater interest however is that this ad-hoc policy generates 89.47% of the welfare gains obtained through the planner's solution (which itself yields a 13.11% increase in real income relative to the benchmark). Both the quasi-

Table 2: Benchmark calibration: summary across experiments

	Benchmark	Quasi-Efficient	Fully Efficient
Avg. Net Output	1	1.1175	1.1311
SD Net Output	1.26	1.21	1.16
Avg. u	0.08	0.13	0.15
$Corr(u, z)$	-0.51	-0.58	-0.18
Avg. MW	1.60	8.35	–
SD MW	0.00	1.09	–

SD is Standard Deviation. Average net output is normalized to unity in the benchmark.

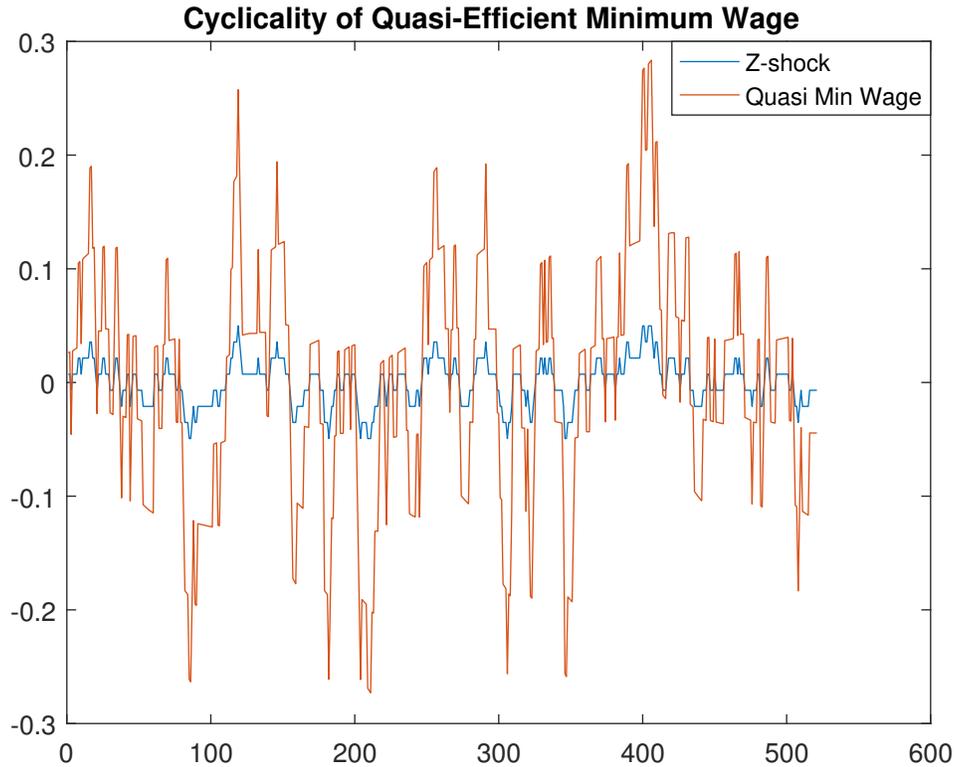
efficient economy and its fully efficient counterpart yield large increases in average output but also feature reduced volatility of net output. This arises from the adjustment of the minimum wage over the cycle. Presumably welfare gains would be even larger under risk averse preferences with an insurance motive that valued the reduction in unemployment volatility. By contrast, the correlation between unemployment and the aggregate shock is slightly larger in the quasi-efficient economy as compared to the benchmark. The comparison across these economies is complicated however by the fact that the distribution of operating firms changes substantially, and with it the sensitivity of vacancy postings to aggregate shocks. The dampening of unemployment (and output) volatility that arises through the policy comes out more clearly in the comparison of the quasi- and fully-efficient economies. The distribution of operating firms is fairly similar across these two experiments, while the correlation between unemployment and the aggregate state falls by two-thirds.

6.1 Flexible Minimum Wage Policy

One of the clear implications of the proposed policy is that minimum wages will respond to aggregate movements in productivity. The resulting pro-cyclical policy that responds to aggregate conditions is quite intuitive. Measured aggregate productivity (the sum of the aggregate shock and average firm productivity) moves slowly and the aggregate shock is quite transitory. Both of these factors motivate a minimum wage policy that limits the responsiveness of firm entry to aggregate shocks. Figure 5 plots a simulated sample of the variation of the minimum wage and aggregate shock process over a ten year period, normalized to percent change with respect to sample average.

The minimum wage policy responds strongly to aggregate shocks, rising and falling over the cycle by nearly 30% of the average. These large responses are necessary to maintain a stable rate of vacancy posting. The pro-cyclical minimum wage policy is reflected in pro-cyclical gains to flexibility, which is not surprising given the logic of the policy itself. From an

Figure 5:



efficiency standpoint, the cost of excessive entry of low productivity firms in booms is higher than to little entry in busts. Our finding contrasts with a current minimum wage policy that is not only sluggish but, if anything, weakly counter-cyclical, adjusting the minimum wage upwards in recessions.

7 Extensions

7.1 Different time horizon

In the benchmark calibration and results reported above, the model runs at weekly frequency, implying that the minimum wage adjusts every week. While there is no obvious technological impediment to operating at such a time frame, we evaluate the performance of the policy operating at monthly, quarterly and annual frequencies. In our simulations, this simply implies the minimum wage is “sticky” in the sense of any other price; it may be updated every 4, 12, or 52 weeks to reflect the current aggregate state of the economy and is constant otherwise. While the policy function is the same across all three economies, the policies

themselves will not converge during weeks in which they all reset. This is because the relative lags in each simulation lead to different paths of aggregate output and unemployment, and hence different states of the world to which the policy responds.

Our simulations suggest that, at least up to a year, the frequency of operation is relatively unimportant. Compared to gains of 11.75% from our benchmark implementation at a weekly frequency, the monthly simulation yields a gain of 11.74%, the quarterly a gain of 11.71% and the annual a gain of 11.64%. While the size of welfare gains is monotonically decreasing, they are quantitatively small compared to the overall welfare gains obtained.

7.2 Aggregate U.S. Labor Market

While our benchmark calibration focuses on a more narrow labor market, the models closest to ours consider the entire U.S. economy. To compare our model to those in the literature, we also perform a calibration exercise with moments from the aggregate U.S. economy. Results are in Table 3. Measures of finding rates $f = 0.139$, market tightness $\theta = 0.634$, proportional posting cost $s = 0.583$, elasticity of wages to productivity $w_{ep} = 0.45$ and the autocorrelation and standard deviation of aggregate output (0.76 and 0.013, respectively) are taken directly from [Hagedorn and Manovskii \(2008\)](#). Moments of the wage distribution and the fraction of earners at the minimum wage are computed using data from CPS-MORG and [Vaghul and Zipperer \(2016\)](#) over the period 1990-2016.

Table 3: Aggregate U.S. Labor Market

	Meaning	Value	Target	Model	Data
s	Posting cost	70.34	$s/(\hat{p} + \bar{z})$	0.583	0.583
κ	Match fun	0.39	v/u	0.634	0.634
\underline{w}	Min wage	2.65	Frac of earners at \underline{w}	0.050	0.051
σ_p	$\text{var}(\log(p))$	1.15	w_{90}	16.42	16.42
μ	mean $\log(p)$	2.93	w_{50}	5.99	7.25
b	Non-market	-99.94	w_{10}	3.2	3.6
\bar{z}	$E[z]$	-145.64	Finding Rate	0.139	0.139
α	Barg. power	0.05	Elast. wages to p	0.44	0.45
ρ	Autocorr z	0.9401	Autocorr Y/L (Q)	0.77	0.76
σ_z	SD z	0.003	SD Y/L (Q)	0.013	0.013

The model is fairly successful in reproducing the targeted moments, and yields parameter values that are quite close to many of those in [Hagedorn and Manovskii \(2008\)](#) in spite of having firm heterogeneity. One parameter of particular interest is worker bargaining power α . This, along with the matching function parameter κ and market tightness θ , are what

determine labor market efficiency (or distance to it). Since θ can be directly observed along with job creation, κ can be estimated directly from the data given functional form assumptions. This leaves the value of α as the main subject of debate. In our calibration exercises, α plays a crucial role in determining the elasticity of wages to aggregate productivity. This, to our eyes, suggests that the cyclical responsiveness of wages carries important information regarding the share of surplus captured.

8 Conclusion

In this paper we demonstrate that the minimum wage can increase welfare, even if redistributive motives are not considered. We show how to use mechanism design to find the optimal minimum wage when the policy-maker has a limited information set, and we have proposed a simple, quasi-efficient policy to set it in a dynamic environment with shocks. We find that the minimum wage should be pro-cyclical.

The minimum wage increases welfare because in our environment it curtails the externality that firms create by posting vacancies. This externality is present in all the quantitative DMP models that do not impose a form of Hosios condition that we could find. However, there are not that many studies of this type. Hence, further work trying to grasp the magnitude of inefficiently high firm entry is crucial.

The point of mechanism design is precisely to achieve efficiency in problems where information asymmetries are present. The set of tools that mechanism design has provided is extensive, and we believe that our take on using a well-known mechanism is also a valuable line for future research. In particular, exercises on the spirit of the quasi-efficient policy, which does not implement the full-efficient allocation but gets close to it, can yield a novel array of quasi-optimal policies.

Last, we find that the minimum wage is pro-cyclical. A planner prefers to prevent entry, hence unemployment, from responding strongly to transitory aggregate shocks, even in the presence of risk-neutral agents. To the best of our knowledge, this is the first paper that studies how the minimum wage should vary over business cycles. We think that further research scrutinizing the cyclicity of the optimal minimum wage is needed.

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A Appendix

A.1 Adding elements to the static problem

In this Appendix we add non-market value for workers, search costs, multiple firms and multiple workers to the static model from Section 3, and show that the resulting minimum wage equation is unchanged.

A.1.1 Adding non-market value for workers

When we add the possibility for workers having non-market value, we need to take into account that if a worker is not matched, she still receives a flow value of non-market activity b . One way of thinking about this element is that the non-market value changes the problem from before by adding a “non-market firm” ready to hire the two workers in case that they do not get matched with a firm — if they do so, they create value $p_{nm} = b$. Hence, a non-market value includes the value created when vacancies are not filled. Returning to firm 3, in this environment the baseline value created if she does not participate is

$$\frac{M(2,2)}{2}(p_1 + p_2) + (2 - M(2,2))b = M(2,2) \left(\frac{p_1 + p_2}{2} - b \right) + 2b.$$

Similarly, the value of other firms if firm 3 participates is given by

$$M(2,3) \left(\frac{p_1 + p_2}{3} - b \right) + 2b.$$

Combining the two expressions from before, we get that the VCG price for firm 3 is given by

$$w_3^{VCG} = \frac{3}{M(2,3)} \times \left(M(2,2) \left(\frac{p_1 + p_2}{2} - b \right) - M(2,3) \left(\frac{p_1 + p_2}{3} - b \right) \right), \quad (20)$$

which, for the case of $b = 0$, gives exactly equations (4). The expression for the VCG price for firms 1 and 2 are the same as equation (20) up to appropriate changes.

As before, there is no reason why p_3 should be higher or lower than w_3^{VCG} , and forcing firm 3 to pay at least w_3^{VCG} is a sufficient condition to guarantee efficiency in the model.

A.1.2 Adding many firms, many workers, and search costs

We now generalize the previous economy by having m workers, with a flow utility from non-market activities b , and n firms, with productivities ordered according to $p_1 > p_2 > \dots > p_n > b$. When a firm posts a job offer, she incurs search cost s . As before, there is a matching function, $M(m, n)$, that determines how many matches happen between m firms posting and n workers searching.

The value generated by other firms if firm i does not participate is

$$\frac{M(m, n-1)}{n-1} \left(\sum_{\forall j/i} p_j \right) + (m - M(m, n-1))b,$$

and the value generated by other firms if firm i decides to participate is

$$\frac{M(m, n)}{n} \left(\sum_{\forall j/i} p_j \right) + (m - M(m, n))b.$$

Again, combining both expressions we can compute the VCG price, which is

$$w_i^{VCG} = \frac{n}{M(m, n)} \left(M(m, n-1) \left\{ \frac{1}{n-1} \sum_{\forall j/i} p_j - b \right\} - M(m, n) \left\{ \frac{1}{n} \sum_{\forall j/i} p_j - b \right\} \right). \quad (21)$$

Note that equation (21) is a direct generalization of equation (20) for the case of many firms and workers. Note that the job posting cost does not appear in the expression. As before, firm i cannot affect w_i^{VCG} , implying, again, that setting the minimum wage according to $\hat{w} = \max\{w_i^{VCG}\}$ induces truth-telling in this environment.

A.2 Proof of Lemmas 1 and 2

The intuition from the VCG mechanism is to charge the boundary firm so that her gain is exactly equal to the social gain from her entry. The welfare function from having all firms with productivity $p \geq p^*$ operating, in the case of a continuum of firms is

$$\Omega(p^*) = M(m, \bar{J}(p^*)) \left[\frac{1}{\bar{J}(p^*)} \int_{p^*}^{\bar{p}} x dJ(x) - b \right] - \bar{J}(p^*)s, \quad (22)$$

where $\bar{J}(p^*) = 1 - J(p^*)$. The first term is the number of matches multiplied by the average gain over unemployment, and the second term is the search costs incurred. Notice that with only a fraction $\bar{J}(p^*)$ firms participating, the truncated CDF is $(J(x) - J(p^*)) / \bar{J}(p^*)$, yielding a PDF of $J'(x) / \bar{J}(p^*)$.

To derive the optimal minimum wage in the continuum of firms case, we compute the per worker wage bill of the firm that controls a positive mass of productivities next the boundary — using the discrete version of the policy — and take the limit of the mass go to zero.

Consider firm with positive mass $(p^*, p^* + \epsilon)$, where p^* is the boundary productivity. Then, dividing the total wage bill that this firm pays by the mass of productivities that this firm controls, $\bar{J}(p^*) - \bar{J}(p^* + \epsilon)$, we can obtain the “per productivity” wage bill. After taking the limit of ϵ to zero we obtain the wage charged to the firm in the boundary.

The total wage bill charged to the boundary firm in the discrete example — which we get using equation (6) — is

$$\hat{w} = \frac{\bar{J}(p^*)}{M(u, \bar{J}(p^*))} \left[M(u, \bar{J}(p^* + \epsilon)) \left\{ \frac{1}{\bar{J}(p^* + \epsilon)} \int_{p^* + \epsilon}^{\bar{p}} xdJ(x) - b \right\} - M(u, \bar{J}(p^*)) \left\{ \frac{1}{\bar{J}(p^*)} \int_{p^* + \epsilon}^{\bar{p}} xdJ(x) - b \right\} \right].$$

Note that profits of this firm can be written as

$$\pi(p^*, p^* + \epsilon) = \frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)} \left[\int_{p^*}^{p^* + \epsilon} xdJ(x) - w \right] - \int_{p^*}^{p^* + \epsilon} sdJ(x). \quad (23)$$

Combining equations (23) and (23) we get exactly $\Omega(p^*) - \Omega(p^* + \epsilon)$. Hence, using the minimum wage makes the marginal firm earn (in total) the net social gain of her entry. The per-firm net social gain is $(\Omega(p^*) - \Omega(p^* + \epsilon))/(\bar{J}(p^*) - \bar{J}(p^* + \epsilon))$.

To find the associated per-worker wage being charged to this firm, $w(\epsilon) = \hat{w}/(\bar{J}(p^*) - \bar{J}(p^* + \epsilon))$, we can write

$$\begin{aligned} w(\epsilon) &= \frac{\bar{J}(p^*)}{M(u, \bar{J}(p^*))} \left[M(u, \bar{J}(p^* + \epsilon)) \left\{ \frac{1}{\bar{J}(p^* + \epsilon)} \int_{p^* + \epsilon}^{\bar{p}} xdJ(x) - b \right\} \right. \\ &\quad \left. - M(u, \bar{J}(p^*)) \left\{ \frac{1}{\bar{J}(p^*)} \int_{p^* + \epsilon}^{\bar{p}} xdJ(x) - b \right\} \right] [\bar{J}(p^*) - \bar{J}(p^* + \epsilon)]^{-1} \\ w(\epsilon) &= \frac{\bar{J}(p^*)}{M(u, \bar{J}(p^*))} \left[\left\{ \frac{M(u, \bar{J}(p^* + \epsilon))}{\bar{J}(p^* + \epsilon)} - \frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)} \right\} \int_{p^* + \epsilon}^{\bar{p}} xdJ(x) \right. \\ &\quad \left. - \{M(u, \bar{J}(p^* + \epsilon)) - M(u, \bar{J}(p^*))\} b \right] [\bar{J}(p^*) - \bar{J}(p^* + \epsilon)]^{-1}. \end{aligned}$$

Taking limits as $\epsilon \rightarrow 0^+$ using L'Hôpital's rule

$$\lim_{\epsilon \rightarrow 0} w(\epsilon) = \frac{\bar{J}(p^*)}{M(u, \bar{J}(p^*))} \left[\frac{\lim_{\epsilon \rightarrow 0} \frac{M(u, \bar{J}(p^* + \epsilon))}{\bar{J}(p^* + \epsilon)} - \frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)}}{\lim_{\epsilon \rightarrow 0} \frac{\bar{J}(p^*) - \bar{J}(p^* + \epsilon)}{\int_{p^* + \epsilon}^{\bar{p}} xdJ(x)}} - \frac{\lim_{\epsilon \rightarrow 0} M(u, \bar{J}(p^* + \epsilon)) - M(u, \bar{J}(p^*))}{\lim_{\epsilon \rightarrow 0} \bar{J}(p^*) - \bar{J}(p^* + \epsilon)} b \right],$$

we get that

$$w(\epsilon) = \frac{\bar{J}(p)}{M(u, \bar{J}(p))} \left[\frac{\int_{p+\epsilon}^{\bar{p}} xdJ(x)}{\bar{J}(p)} \left(\frac{M(u, \bar{J}(p))}{\bar{J}(p)} - M_2(u, \bar{J}(p)) \right) + M_2(u, \bar{J}(p)) b \right].$$

Hence, the wage charged to the marginal firm is

$$w_{\text{marginal}} = \frac{\bar{J}(p^*)}{M(u, \bar{J}(p^*))} \left[\frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)} \mu(p^*) - M_2(u, \bar{J}(p^*)) \{\mu(p^*) - b\} \right], \quad (24)$$

where $\mu(p^*) = \int_{p^*}^{\bar{p}} x dJ(x) / \bar{J}(p^*)$.

We can easily verify that this is also the wage that must be charged for the firms with the (efficient) boundary productivity are just willing to enter. Using the social planner function $\Omega(p^*)$ from above, we can write

$$\Omega'(p^*) = M_2(u, \bar{J}(p^*)) \bar{J}'(p^*) [\mu(p^*) - b] + M(u, \bar{J}(p^*)) \mu'(p^*) - \bar{J}'(p^*) s. \quad (25)$$

Since firms pay a search cost s , we want the boundary productivity's expected gains (absent search costs) to equal

$$\frac{M_2(u, \bar{J}(p^*)) \bar{J}'(p^*) [\mu(p^*) - b] + M(u, \bar{J}(p^*)) \mu'(p^*)}{\bar{J}'(p^*)} = s, \quad (26)$$

which simplifies to

$$M_2(u, \bar{J}(p^*)) [\mu(p^*) - b] + \frac{M(u, \bar{J}(p^*))}{\bar{J}'(p^*)} \mu'(p^*) - s = 0. \quad (27)$$

The optimal minimum wage, w^{optimal} , has to satisfy that profits of the marginal firm coincide with equation (27). The profits of the marginal firm, after paying the optimal minimum wage, are

$$\pi(p^*) = \frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)} [p^* - w^{\text{optimal}}] - s. \quad (28)$$

Equations (27) and (28) are equal if the optimal minimum wage satisfies

$$w^{\text{optimal}} = \frac{\bar{J}(p^*)}{M(u, \bar{J}(p^*))} \left(\frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)} \mu(p^*) - M_2(u, \bar{J}(p^*)) [\mu(p^*) - b] \right). \quad (29)$$

Note that equations (24) and (29) are exactly the same, which implies that the minimum wage derived from the policy coincides with the minimum wage that induces efficiency in the market. Moreover, both expressions are equal to equation (9), which proves the result of the Lemma. ■

A.3 Proof of Theorem 4

First, consider all possible bidding firms given any particular $S \in \mathcal{S}$. Note that because $t_{\Gamma}(\gamma, S)$ depends only on the average reported valuation, no individual firm has an incentive to misreport (firms are atomistic). Hence for any S , valuations will be truthfully reported, so $\gamma(p) = p$.

Now consider the unique strategy profile S^* such that the planners' solution is efficiently implemented. In this case, $\delta^*(p) = 0 \forall p < p^*$ and $\delta^*(p) = 1, \forall p \geq p^*$ and $\gamma^*(p) = p \forall p$.

Moreover, under truthful reporting and efficient entry, the proposed minimum wage \hat{w} implies that $\pi(p^*) = 0$, which is a direct consequence of Lemma 1.

Finally, given that the wage charges the marginal firm such that their profits are equal to the social value of their entry, notice that no firm with $p < p^*$ has an incentive to enter, since their expected profits will be negative. ■

A.4 Proof of Lemma 6

To prove the result, it is useful to first characterize some parts of the problem. In particular, the derivatives of the mass of operating firms with respect to the marginal firm is

$$\frac{d\bar{J}(p^*)}{dp^*} = -\frac{dJ(p^*)}{dp^*},$$

and the derivative of the expected entry productivity with respect to the marginal firm is

$$\begin{aligned} \frac{dE(p|p > p^*)}{dp^*} &= \frac{-p^* \bar{J}(p^*) \frac{dJ(p^*)}{dp^*} + \int_{p^*}^{\hat{p}} x dJ(x) \frac{dJ(p^*)}{dp^*}}{\bar{J}(p^*)^2}, \\ &= -\frac{(E(p|p > p^*) - p^*) d\bar{J}(p^*)}{\bar{J}(p^*) dp^*}. \end{aligned}$$

The first order condition that the planner takes from problem (16) with respect to p^* is

$$\begin{aligned} 0 &= -s \frac{d\bar{J}(p^*)}{dp^*} - \beta \Omega_1(u', \hat{p}', z') \left(M_2(u, \bar{J}(p^*)) \frac{d\bar{J}(p^*)}{dp^*} \right) \\ &\quad + \beta \Omega_2(u', \hat{p}', z') \left(M_2(u, \bar{J}(p^*)) \frac{(E(p|p > p^*) - \hat{p}')}{1 - u'} - \frac{M(u, \bar{J}(p^*)) (E(p|p > p^*) - p^*)}{\bar{J}(p^*) (1 - u')} \right) \frac{d\bar{J}(p^*)}{dp^*}. \end{aligned}$$

Since $\frac{d\bar{J}(p^*)}{dp^*}$ appears everywhere, the problem is

$$\begin{aligned} 0 &= -s - \beta \Omega_1(u', \hat{p}', z') M_2(u, \bar{J}(p^*)) \\ &\quad + \beta \Omega_2(u', \hat{p}', z') \left(M_2(u, \bar{J}(p^*)) \frac{(E(p|p > p^*) - \hat{p}')}{1 - u'} - \frac{M(u, \bar{J}(p^*)) (E(p|p > p^*) - p^*)}{\bar{J}(p^*) (1 - u')} \right). \end{aligned}$$

The envelope condition for \hat{p} is

$$\Omega_2(u, \hat{p}, z) = (1 - u) + \beta \frac{(1 - \delta)(1 - u)}{(1 - u')} \Omega_2(u', \hat{p}', z').$$

Again, using consecutive periods we get

$$\begin{aligned}
\Omega_2(u, \hat{p}, z) &= (1 - u) \\
&\quad + \beta \frac{(1 - \delta)(1 - u)}{(1 - u')} \left((1 - u') + \beta \frac{(1 - \delta)(1 - u')}{(1 - u'')} \Omega_2(u'', \hat{p}'', z'') \right) \\
&= (1 - u) + \beta(1 - \delta)(1 - u) \\
&\quad + \beta^2 \frac{(1 - \delta)^2(1 - u)}{(1 - u'')} \Omega_2(u'', \hat{p}'', z'') \\
&= (1 - u) + \beta(1 - \delta)(1 - u) + (\beta(1 - \delta))^2(1 - u) \\
&\quad + \beta^3 \frac{(1 - \delta)^3(1 - u)}{(1 - u''')} \Omega_2(u''', \hat{p}''', z''') \\
&= (1 - u) \sum_{s=0}^{\infty} (\beta(1 - \delta))^s \\
&= \frac{1 - u}{1 - \beta(1 - \delta)}.
\end{aligned}$$

The envelope condition for u is

$$\begin{aligned}
\Omega_1(u, \hat{p}, z) &= -(z + \hat{p}) + b + \beta(1 - \delta - M_1(u, \bar{J}(p^*))) \Omega_1(u', \hat{p}', z') \\
&\quad + \beta \frac{(1 - u')}{1 - \beta(1 - \delta)} \frac{(\hat{p}' - \hat{p})(1 - \delta) + M_1(u, \bar{J}(p^*))(E(p|p > p^*) - \hat{p}')}{1 - u'}.
\end{aligned}$$

Define it as

$$\begin{aligned}
\Omega_1(u, \hat{p}, z) &= A + B\Omega_1(u', \hat{p}', z') \\
\Omega_1(u, \hat{p}, z) &= A + B(A' + B'\Omega(u'', \hat{p}'', z'')) \\
\Omega_1(u, \hat{p}, z) &= A + BA' + BB'(A'' + B''\Omega(u''', \hat{p}''', z''')) \\
\Omega_1(u, \hat{p}, z) &= A^{(0)} + \sum_{i=1}^{\infty} \prod_{j=0}^{i-1} B^{(j)} A^{(i)},
\end{aligned}$$

where

$$A^{(i)} = -z^{(i)} - \hat{p}^{(i)} + b + \beta \frac{(\hat{p}^{(i+1)} - \hat{p}^{(i)})(1 - \delta) + M_1(u^{(i)}, \bar{J}(p^*)^{(i)})(E(p|p > p^*)^{(i)} - \hat{p}^{(i+1)})}{1 - \beta(1 - \delta)},$$

$$B^{(i)} = \beta(1 - \delta - M_1(u^{(i)}, \bar{J}(p^*)^{(i)})),$$

and $^{(i)}$ denotes the variable i periods ahead.

Combining the two envelope conditions into the first order condition, we get that

$$\begin{aligned}
0 = & -s - \beta M_2(u, \bar{J}(p^*)) \left(-z' - \hat{p}' + b + \beta \frac{(\hat{p}'' - \hat{p}')(1 - \delta) + M_1(u, \bar{J}(p^*))'(E(p|p > p^*)' - \hat{p}'')}{1 - \beta(1 - \delta)} \right) \\
& - \beta M_2(u, \bar{J}(p^*)) \sum_{i=2}^{\infty} \prod_{j=1}^{i-1} \beta(1 - \delta - M_1(u^{(j)}, \bar{J}((p^*)^{(j)}))) \left(\begin{array}{l} -z^{(i)} - \hat{p}^{(i)} + b \\ + \beta \frac{(\hat{p}^{(i+1)} - \hat{p}^{(i)})(1 - \delta)}{1 - \beta(1 - \delta)} \\ \left(M_1(u^{(i)}, \bar{J}(p^*)^{(i)}) \right. \\ \left. \times (E(p|p > p^*)^{(i)} - \hat{p}^{(i+1)}) \right) \\ + \beta \frac{\quad}{1 - \beta(1 - \delta)} \end{array} \right) \\
& + \frac{\beta}{1 - \beta(1 - \delta)} \left(M_2(u, \bar{J}(p^*)) (E(p|p > p^*) - \hat{p}') - \frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)} (E(p|p > p^*) - p^*) \right).
\end{aligned}$$

Rearranging we get the desired result. ■

A.5 Heterogeneous workers

In this appendix we show how the model can be extended to account for worker's heterogeneity. Assume that the mass 1 of workers in the economy have an efficiency h distributed according to some distribution H . We keep the assumption that all workers are joining the search market.

The value for worker with efficiency h of being unemployed, U , is given by

$$U(z, u, h) = b + \beta E \left[\lambda(z', u', h) \int_{p^*(z', u')} W(x, z', u', h) dJ(x) + (1 - \lambda(z', u', h)) U(z', u', h) \right], \quad (30)$$

in what is the counterpart of equation (11). Notice that the only way in which the value of being unemployed is affected by workers' efficiency is by affecting the value of being employed. In turn, the value of being employed at firm p , is given by

$$W(p, z, u, h) = \max\{\hat{w}, w(p, z, u, h)\} + \beta E [(1 - \delta)W(p, z', u', h) + \delta U(z', u', h)] \quad (31)$$

in what is the counterpart of equation (12).

The value of an operating firm with productivity p , matched with a worker with efficiency h , is

$$\Pi(p, z, u, h) = z + p + h - w(p, z, u, h) + \beta(1 - \delta)E [\Pi(p, z', u', h)], \quad (32)$$

the counterpart of equation (13). Last, the free entry condition pinning down the marginal productivity p^* is

$$\frac{M(u, \bar{J}(p^*(u, z)))}{\bar{J}(p^*(u, z))} \beta E [\Pi(p^*(u, z), u, z, h)] = s, \quad (33)$$

the counterpart of equation (14).

This version of the model delivers the same allocations as the model without worker heterogeneity as long as the marginal firm is the same. The marginal firm can be computed implicitly as

$$\frac{M(u, \bar{J}(p^*))}{\bar{J}(p^*)} \beta \left(\frac{p^* + E[h]}{1 - \beta(1 - \delta)} + E \left[\sum_{s=0}^{\infty} (\beta(1 - \delta))^s (z^{(s)} - \max\{w^{(s)}, \hat{w}\}) \right] \right) = s. \quad (34)$$

Note that worker heterogeneity in this model only appears in the numerator of the first term in brackets of equation (34), when it its counterpart without worker heterogeneity, (15, the term is not there.

The two models deliver the same cut-off, and hence the same allocations. Let \hat{p} be firm productivity in the model without worker heterogeneity and p firm productivity in the model with worker heterogeneity. Then, set $\hat{p} = p + E[h]$, and the two models imply the same allocations.