Nonlinear Gravity

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July 2016

Abstract

In constant elasticity of substitution (CES) trade models, the elasticity of import intensity to trade costs is constant while in non-CES models, it is a function. We provide a general formula for this function without making functional form assumptions on the utility function and allowing for quite general production environments. We show how to use the formula to measure welfare gains and to compare them between CES and non-CES models. In a quantitative application we find that more closed countries and countries with similar patterns of production and consumption across sectors have gains larger than those implied by CES models.

1 Introduction

Most well-known models of international trade feature a constant elasticity of subsitution (CES) import demand system, are analytically tractable, and have welfare gains from trade that are easy to compute. These models include Krugman (1980), Eaton and Kortum (2002), and Melitz (2003) among others. The tractability of these models is largely due to the CES demand structure, which is why we refer to them as CES models. Other, increasingly popular models of international trade feature non-CES import demand systems and can match many

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different patterns of the data. These models include Markusen (1986), Fieler (2011), Caron, Fally and Markusen (2014), and Simonovska (2015) among others. Their ability to match international trade patterns such as the composition of trade flows for different income levels, the elasticity of substitution across sectors or the elasticity of price with respect to per-capita income—patterns that CES models cannot match—is due to their non-CES demand structure, which is why we refer to them as non-CES models. But non-CES models have an important caveat: they are (in general) not analytically tractable, which makes them difficult to use to perform comparative statics within the model as well as welfare comparisons across different models. In this paper we show how to overcome this caveat: we derive a formula for the welfare gains from trade without imposing any particular utility function. We show how to use this formula i) to establish what characteristics of a model determine the welfare gains from trade; ii) to compare the welfare gains implied by a non-CES model to those implied by a CES model; and iii) to recover welfare gains implied by empirical papers that allow for non-CES demand structures. The latter is done by means of an example; with it, we find that more open countries have lower marginal gains from trade and countries with similar patterns of production and consumption across sectors have larger gains from trade.

Our formula represents an extension of the welfare gains formula derived by Arkolakis, Costinot and Rodriguez-Clare (2012, hereafter referred to as ACR): theirs measures the welfare gains from trade in CES environments, while ours measures the welfare gains from trade in non-CES environments. Since our formula is a generalized version of their formula, the two formulae predict the same welfare gains from trade in CES environments. The assumptions behind our result are those of ACR, except that instead of a CES import demand system, we only require the household to have a strictly increasing, strictly concave and twice continuously differentiable utility function. All the other assumptions are the same: there is a single factor of production in each country that is in fixed supply; aggregate profits are proportional to factor payments; the model is static; final goods are composed of intermediates from many countries that are aggregated using a constant returns to scale—though not necessarily CES—aggregator.

Our welfare gains equation is derived in two steps. First, we show how to solve for the aggregate trade elasticity when preferences are non-homothetic and production is non-CES. As we show in the body of the paper, the trade elasticity consists of a weighted average of all household and sector-level trade elasticities, with the weights depending on the fraction of total import and the relative share of domestic expenditure on the sector, plus a term that corrects all the reallocation of expenditures across sectors. Second, we show that in this environment, like in ACR, the elasticity of real income with respect to one minus import intensity is equal to the reciprocal of the trade elasticity. These two results together provide an easy means of measuring gains from trade.

CES models exhibit a gravity form: the traditional (log-log) linear gravity model employed by the empirical trade literature (see Anderson (1979) and Bergstrand (1985)) is correctly specified within CES models. In these models, the gravity form is driven by the simple relationship between import demand and trade costs. This simple relationship allows to measure the gains from reducing trade costs easily: they are equivalent to increasing the import intensity. As demonstrated in ACR, the marginal change in real income from a marginal change in import intensity (which we call the welfare elasticity) is:

$$\frac{\partial \log(W)}{\partial \log(\lambda_{00})} = \frac{1}{\varepsilon_T} \tag{1}$$

where W is real income at the observed level of trade, λ_{00} is one minus the ratio of imports to domestic gross output and ε_T is the elasticity of import intensity to trade costs (hereafter we refer to this as the trade elasticity), an object commonly estimated in the empirical literature using linear gravity equations.

The gravity equation in trade is a powerful tool that has greatly improved the understanding of trade patterns. Gravity equations were developed when researchers' data consisted of trade flows, income, distance between countries, and other aggregate country level characteristics. New, more detailed data-sets allow to strengthen the estimation of the gravity equation by letting objects that are typically assumed constant—for instance, the trade elasticity—to have more flexibility. Pioneering this new research, Novy (2013) estimates a translog trade elasticity function and, as a result, finds that trade costs have heterogeneous impact across country pairs. Typically, non-CES models deliver a non-constant trade elasticity. We show in Section 6 how the elasticity term varies across countries using the non-CES model of Caron, Fally and Markusen (2014). A non-constant trade elasticity means that the underlying trade model has to be different than CES. Moving to a non-CES environment means that ε_T is no longer a constant, but a function that may vary across countries or as trade costs change. Therefore, the results from ACR—which hinge on a constant elasticity—no longer apply. In order to measure the gains from trade, we first solve for the trade elasticity function ε_T in non-CES environments, and then show that the trade elasticity and the welfare elasticity are the reciprocal of each other. Hence, in non-CES environments:

$$\forall \tau : \frac{\partial \log(W(\tau))}{\partial \log(\lambda_{00}(\tau))} = \frac{1}{\varepsilon_T(\tau)} \tag{2}$$

where the functional dependence of ε_T on τ is meant to emphasize that the trade elasticity changes as trade costs change. Then, given the formula for the trade elasticity, equation (2) provides a means of measuring gains from trade in non-CES environments.

Equation (2) implies that trade elasticities need not be constant across countries. In Section 4 we provide many examples where country-level trade elasticities varies: countries have different sectoral composition of production and imports, countries differ in the fraction of firms that export, or countries have different levels of trade openness, among others. The margins that affect the aggregate trade elasticity depend on the particular model being considered. This is in sharp contrast to CES models, where trade elasticities are constant across countries and are independent of country characteristics.

Equation (2) also implies that trade elasticities may change as costs change; for instance, trade elasticities near autarky may be very different from trade elasticities at observed levels of trade. In order to compute the welfare gains from trade, we need to integrate equation (2) between autarky and free trade. Let W be real income at observed levels of trade and let W^{AUT} be real income under autarky, then, the increase in real income is given by:[1]

$$\log\left(\frac{W}{W^A}\right) = \frac{1}{\varepsilon_T(\tau)}\log(\lambda_{00}) + \int_{\log(\tau)}^{\infty} -\frac{\log(\lambda_{00})}{\varepsilon_T(\tau)^2}\frac{\partial \varepsilon_T(\tau)}{\partial \log(\tau)} d\log(\tau) \tag{3}$$

The first term of equation (3) coincides with the gains from trade derived by ACR. If the trade elasticity is a constant, the second term of equation (3) (which includes the derivative of this elasticity) is zero and the ACR result is recovered. If the trade elasticity is not constant but it is a monotone function of trade costs, then the second term in equation (3) has the same sign as the derivative of the trade elasticity with respect to trade costs. As a result, if the trade elasticity at observed levels of trade is higher than near autarky, then gains near observed

[1] We are considering moving from autarky to observed trade because that is the baseline exercise in ACR.

levels of trade are higher than they are near autarky and vice-versa. Since we can solve for the explicit trade elasticity function, we can sign the derivative of the elasticity with respect to trade costs relatively easily. Some of the examples in Section 4 are based on models that cannot be solved analytically, yet the trade elasticity can be solved in closed form, and the sign of the derivative of the trade elasticity with respect to trade costs can be determined easily.

In Section 6 we show how to use estimates from empirical papers to recover implied welfare gains from trade using our formula. We consider the model in Caron, Fally and Markusen (2014), which we refer to hereafter as CFM, because it is consistent with many patterns of trade that are not matched by CES models, and cannot be solved in closed form. CFM features a household with non-homothetic preferences over goods from many sectors, making sectors have different income elasticities. Production is as in Eaton and Kortum (2002), but each sector is allowed to have a different shape parameter on the distribution of productivity draws, generating different sector-level trade elasticities. We bring the CFM estimates to our formula to solve for the aggregate trade elasticity and gains from trade. Country-level aggregate trade elasticities vary considerably across the 118 countries in the sample: the average trade elasticity is -7.5, which coincides with the average trade elasticity in Anderson and Wincoop (2004), but range from -4.42 (Peru) to -11.37 (Luxembourg). For the United States, the increase in real income from autarky to observed trade is 1.71%, almost doubling the 0.99% that would be implied using the ACR formula and a trade elasticity of -7.5. For China, gains are 2.73%, but the ACR formula implies gains of 1.42%. We show that the aggregate trade elasticities (and, hence, gains from trade) vary systematically with country characteristics. The first characteristic is import penetration, as demonstrated in Figure 1: countries that are more open to trade have lower trade elasticities—and lower marginal gains from trade—than less open countries. The second characteristic is sectoral composition: countries with similar patterns of production and consumption across sectors have larger gains from trade than countries with dissimilar patterns of production and consumption.

Ossa (2015), Melitz and Redding (2014) and Melitz and Redding (2015) also consider deviations from CES trade models and their implications for gains from trade. Ossa (2015) uses an example to show that substantial heterogeneity in sector-level trade elasticities imply that aggregate elasticities understate gains from trade by an intuitive, yet general Jensen's inequality argument. This is quite related to our quantitative exercise except that we can add a second dimension to this argument not present there due to Cobb-Douglas preferences: in response to increases in trade costs countries substitute expenditure away from the most highly imported sectors because they experience the greatest increases in prices. Melitz and Redding (2014) constructs an example with sequential production to show that gains from trade may be unboundedly high, even though gains from trade may be small at the margin. Melitz and Redding (2015) shows how deviations from the Pareto distribution in the Melitz (2003) model generate a non-CES environment and that the aggregate trade elasticity and import penetration ratio are no longer sufficient to characterize gains from trade. Relative to these papers, our first innovation is to have results that apply to a broad class of non-CES models, rather than focusing on examples, and thereby to greatly expand the set of models that can be used to measure gains from trade in a tractable way. Second, our results focus mainly on computing cross-country differences in trade elasticities and marginal gains from trade as a function of observables. We have some results about non-local changes in trade costs, as discussed in Section 5, but this is not our emphasis.

This paper is closely related to the many papers that investigate gains from trade in models with sector-level heterogeneity, including Broda and Weinstein (2006), Ardelean and Lugoskyy (2010), Feenstra and Weinstein (2010) and Blonigen and Soderbery (2010). The strength of this paper is to nest many different environments and demonstrate how to directly relate measured trade elasticities to changes in welfare. It is also closely related to other recent papers that have expanded the results of the original ACR paper, such as Arkolakis et al. (2014) and Allen, Arkolakis and Takahashi (2014). These papers are focused on the relationship between expanded models and the traditional, linear gravity model. In this paper, we consider models with non-homotheticities so that the linear gravity regression is incorrectly specified. In this sense, we view our paper as complementary to those. Lastly, recent work by Adao, Costinot and Donaldson (2015) provides a general framework to map non-parametric estimates of trade responses into gains from trade without having to specify many of the features of the underlying model. Our paper takes the opposite approach, starting with a model and providing an easy means of comparing implied welfare gains across models.

2 Environment

We consider a two country environment where households consume, inelastically supply labor and own firms, and where final good producers purchase intermediate goods from the domestic country and from abroad. The assumption on two countries is for clarity of exposition only; we show how to extend the results to more countries in Appendix A. Throughout the paper, we take the perspective that country 0 is the country of analysis, and country 1 is the rest of the world.

2.1 Household's problem

The household in country 0 has an additively separable utility function and solves the following problem:

$$\begin{aligned} &\max_{\{c(i)\}\geq 0} \int_{i\in\Omega} u(c(i),i)di \\ st: &L_0 \int_{i\in\Omega} p(i)c(i)di = I_0 \equiv w_0 L_0 + \Pi_0 \\ &c(i) \geq 0 \end{aligned} \tag{4}$$

where $c(i)$ is the consumption of good i and $p(i)$ is its price; there is a set Ω of goods produced domestically using domestic and foreign intermediates; w_0 is the wage in country 0, which we make the numeraire, ; L_0 is the population of identical households in country 0, and all of them supply one unit of labor inelastically; Π_0 is the lump sum rebate of all profits generated by intermediate firms in country 0.

In problem (4) the utility function is not specified. We only require the utility to be strictly increasing, strictly concave and twice differentiable. In general, preferences described by these utility functions are non-homothetic. The non-negativity constraint on consumption expenditure implies that there may be an extensive margin in the consumption of different types of goods. The assumption on additively separable utility function is for clarity of exposition only; we show how to extend the results to the non-separable case in Appendix B.

2.2 Production

We assume that each sector has a set of competitive final goods producers. These firms purchase intermediate goods both from the domestic country and from abroad, and aggregate them into the final good in each sector i. The price of the final good incorporates trade costs from imports, as well as the prices of all intermediates.

The problem of the competitive intermediate firm producer is given by:

$$\begin{aligned} p(i) = & \min_{\{x_{0n}(i,j)\}\geq 0} \sum_{n=0}^{1} \int_{j\in\Upsilon_n(i)} \tau_{0n} q_n(i,j) x_{0n}(i,j) dj \\ st: & \quad 1 = F\left(i, \{\{x_{0n}(i,j)\}_{j\in\Upsilon_n(i)}\}_n\right) \end{aligned} \tag{5}$$

where $p(i)$ is the price of good i; $x_{0n}(i,j)$ is a the quantity of variety j used in the production of good i, $q_n(i,i)$ is the price of this good, and $\Upsilon_n(i)$ is the set of varieties available from country n in sector i; following ACR, we assume that the trade cost τ_{0n} is proportional across sectors and throughout the paper we will be considering proportional changes to it.

We make three assumptions about the intermediate good sector. First, F is constant returns to scale, so that the final goods producers generate no profits. Second, the prices of intermediate goods $q_n(i,j)$ are linear in country n wages, and independent of trade costs and wages in other countries. Third, aggregate profits from the production sector in country 0, Π_0, are proportional to the wage in country 0.[2]

In each country, labor markets are cleared by an equilibrium wage. Importantly, wages may differ across countries so that there are nontrivial general equilibrium effects from changes in trade costs.

3 Trade and Welfare Elasticities

In CES models, the elasticity of real income with respect to one minus import penetration is equal to the reciprocal of the elasticity of relative trade shares to trade costs, as in ACR. In this section, we show that this is still true locally in the more general environment described in the previous section, and explicitly characterize the trade elasticity.

[2]Note that zero aggregate profit is a special case of this. This last assumption is an important restriction in that it does not allow us to consider environments that have pro-competitive effects of trade liberalization, which is an important ongoing area of research (see Arkolakis et al. (2014), Holmes, Hsu and Lee (2014), Feenstra (2014), and Edmond, Midrigan and Xu (2015)).

3.1 Characterization of Variable Trade Elasticity

The trade elasticity is defined as:[3]

$$\varepsilon_T \equiv \frac{\frac{\partial \log(\lambda_{01}/\lambda_{00})}{\partial \log(\tau_{01})}}{1+\frac{\partial \log(w_1)}{\partial \log(\tau_{01})}} \tag{6}$$

where $\lambda_{0n} = I_0^{-1}\int_\Omega \lambda_{0n}(i)X_0(i)di$ is the expenditure share of the domestic country on country n, $\lambda_{0n}(i) = {X_{0n}(i)}/{X_0(i)}$ is the expenditure share of the domestic country on country n for good i; $X_0(i) = X_{00}(i)+X_{01}(i)$ is the total expenditure of the domestic country on good i; $X_{0n} = \int_\Omega X_{0n}(i)di$ is the expenditure level of the domestic country on country n, and $X_{0n}(i) = \int_{\Upsilon_n(i)} \tau_{0n}q_n(i,j)x_{0n}(i,j)dj$ is the expenditure level of the domestic country on country n for good i.

Only in the CES case is the trade elasticity a constant. With other types of preferences or production structures, the trade elasticity varies with trade costs. This means that changes in trade costs do not have a constant marginal effect on imports; hence non-CES environments feature a "nonlinear gravity" relationship. Our goal here is to characterize the trade elasticity in non-CES frameworks, but before doing so, we need to characterize two more objects. The first is the sector-level trade elasticity that captures how imports change with trade costs in each sector, which is given by:

$$\rho(i) = \frac{\frac{\partial \log(\lambda_{01}(i)/\lambda_{00}(i))}{\partial \log(\tau_{01})}}{1+\frac{\partial \log(w_1)}{\partial \log(\tau_{01})}},$$

The second is the curvature term for each sector, which summarizes how households change expenditure across sectors:

$$\kappa(i) \quad = \quad \begin{cases} 0 & \text{if } c(i)=0 \\ \frac{u'(i)}{c(i)u''(i)} & \text{if } c(i)>0 \end{cases}. \tag{7}$$

These terms, together with expenditure shares by sector and origin, are sufficient to compute the trade elasticity as given in Proposition 1.

[3]To be precise, this is the elasticity of trade with respect to changes in country-level trade costs. The preceding literature omits the denominator in the definition we provide and refers to it as the "partial trade elasticity", in that it abstracts from general equilibrium effects. For the purposes of the counterfactuals considered in this paper, ours is equivalent to the definition in the existing literature. We include the denominator clarity in some proofs. See Novy (2013) for more discussion of the general equilibrium effect.

Proposition 1 *Whenever* $\lambda_{00} \in (0,1)$*:*

$$\begin{aligned}\varepsilon_T = \quad & -\int_\Omega (1+\kappa(i))\,\frac{\omega_{01}(i)\omega_{00}(i)}{\omega_0(i)}di + \int_\Omega \rho(i)\frac{\omega_{01}(i)\omega_{00}(i)}{\omega_0(i)}di \\ & + \int_\Omega (1+\kappa(i))\,\omega_{01}(i)di\left[\frac{\int_\Omega \kappa(i)\omega_{00}(i)di}{\int_\Omega \kappa(i)\omega_0(i)di}\right].\end{aligned} \tag{8}$$

where

$$\omega_{0j}(i) = \frac{X_{0j}(i)}{X_{0j}}$$

$$\omega_0(i) = \frac{X_{00}(i) + X_{01}(i)}{X_{00} + X_{01}}$$

The proof of Proposition 1 is in Appendix C. The formula for the trade elasticity, equation (8), describes how the share of expenditure in imports changes when there is a change in trade cost, taking into account all the general equilibrium effects. There are three parts: the household elasticity, which depends on $\kappa(i)$, and summarizes how the household reduces consumption in each sector when prices in those sectors rise due to higher trade costs;[4] the production elasticity, which depends on $\rho(i)$, and summarizes how import content changes in each sector when prices in those sectors rise due to higher trade costs; and the reallocation term, which shows how the household reallocates its budget across sectors to satisfy its budget constraint. Adding the three terms up shows that the trade elasticity consists of a weighted average of all household and sector-level trade elasticities, with the weights depending on the fraction of total import and the relative share of domestic expenditure on the sector, plus a term that corrects all the reallocation of expenditures across sectors.

Note that the first and third term added together can be interpreted as the effect on aggregate trade flows of the reallocation of consumption across sectors in response to a change in trade costs. When trade costs increase, prices increase relatively more in sectors with higher import penetration. Therefore, households shift consumption away from high import penetration sectors toward sectors with lower import penetration, an effect that reduces aggregate imports. This implies a decrease in trade flows that is independent of changes in import penetration within sectors. This type of effect cannot be accounted for by a CES model.

[4] Although the second effect implies that quantities decrease, expenditures need not do so. If $\kappa(i) \in (-1, 0)$, then expenditures on imports (including trade costs) rise with trade costs.

3.2 Empirical Content of Variable Trade Elasticity

Given the characterization of the trade elasticity derived in the last subsection, it is clear that non-CES models typically imply variable trade elasticities. As such, the trade elasticity is no longer characterized by a single number as in the class of models studied in ACR. Therefore, some discussion is warranted on how to apply or measure this variable trade elasticity in the data.

The purpose of this paper is not to develop any novel empirical strategies to measure this variable trade elasticity directly. Rather, the results developed in the previous subsection provide a means of measuring the aggregate trade elasticity in the large number of empirical papers that depart from CES. These include papers that measure sector-level trade elasticities, such as Broda and Weinstein (2006), Feenstra (1994) and Soderbery (2015). Also, we can measure trade elasticities in models with non-homothetic preferences, such as Fieler (2011), Caron, Fally and Markusen (2014) and Markusen (1986). These environments fit into the framework developed so far, and the results of Proposition 1 show how to turn the parameter estimates from those models and empirical strategies into an aggregate trade elasticity. In this regard, we are similar in spirit to ACR, which provided no new empirical strategy to measure the constant trade elasticity, but showed how to use existing empirical estimates to measure gains from trade. In the previous subsection we showed how to measure the aggregate trade elasticity in non-CES models, and in the next subsection we show how to relate them to welfare gains from trade. All of Section 6 is devoted to an application where we use the estimates from Caron, Fally and Markusen (2014) to measure variable trade elasticities for a large number of countries, then measure welfare gains from trade.

3.3 Welfare Elasticity

In this section we show that the trade elasticity from equation (8) is more important than to simply measure how imports change when trade costs change. We show—similarly to how ACR show it in a CES framework—that the trade elasticity in this framework is the reciprocal of the welfare elasticity. Hence, computing welfare gains from trade in non-CES environments can be done using equation (8). This equivalence is true even though the trade elasticity is a function.

In order to show our result, note that in a non-CES environment there is not a perfect price index, which implies that changes in real income are not a direct function of changes in the price level. Instead, changes in real income have to be computed as the compensating variation needed to make the household indifferent between the allocation they receive facing current trade costs and current income, and the allocation they would receive with new trade costs. Denoting real income as W, the dual of the household problem (4) is given by

$$\begin{aligned} I = \quad & \min \int_{i\in\Omega} p(i)c(i)di \\ s.t. : \bar{U} = \quad & \int_{i\in\Omega} u(c(i), i)di \end{aligned} \tag{9}$$

In this setup I is not real income: for a given $\bar{U}$, I is increasing in τ_{01}, since the household would need more units of income to afford the same utility level. Instead, changes in real income are defined as the equivalent loss in income associated with an increase in trade costs. Therefore,

$$\frac{\partial \log(W)}{\partial \log(\tau_{01})} = -\frac{\partial \log(I)}{\partial \log(\tau_{01})} \tag{10}$$

We define the welfare elasticity as the elasticity of real income with respect to changes in expenditure on domestic goods,

$$\varepsilon_W \equiv \frac{\frac{\partial \log(W)}{\partial \log(\tau_{01})}}{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}} = -\frac{\frac{\partial \log(I)}{\partial \log(\tau_{01})}}{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}} \tag{11}$$

in order to show, as we do in Proposition 2, that the trade elasticity, equation (8) and the welfare elasticity, equation (11), are the reciprocal of each other.

Proposition 2 $\forall \lambda_{00} \in (0, 1)$,

$$\varepsilon_W = \frac{1}{\varepsilon_T}$$

The proof of Proposition 2 is in Appendix D. We can now combine the results of Propositions 1 and 2 to solve for gains from trade even in non-CES models.

4 Application to Existing Models

Here we solve several examples to illustrate how to apply Proposition 1 to solve for the trade elasticity in non-CES models. [5]

[5] As demonstrated above, the household's utility function governs substitution across sectors, while the final good producer's problem within each sector determines the tradeoff between imports and domestic intermediates within each sector. An alternative interpretation

4.1 Constant Relative Income Elasticity

Caron, Fally and Markusen (2014) assumes the following utility function for each sector:

$$u(i) = \frac{\alpha(i)\sigma(i)}{\sigma(i)-1} c(i)^{1-1/\sigma(i)}$$

This specification is called "constant relative income elasticity" because the relative income elasticities among goods with different values of σ are fixed.

To compute the aggregate trade elasticity in this environment, we need only take the first and sector derivatives of the sector utility functions:

$$u'(i) = \alpha(i)c(i)^{-1/\sigma(i)}, \quad u''(i) = -\frac{u'(i)}{\sigma(i)c(i)} \implies \kappa(i) = -\sigma(i)$$

This case is particularly simple because the values of κ are simple elasticities. Moreover, notice that the CES case is recovered if the $\sigma(i)$ terms are equal in all sectors. However, it should be noted if there are multiple sectors then $\forall i, \sigma(i) = \sigma$ is not sufficient for a constant aggregate trade elasticity.

4.2 Stone-Geary Preferences

Now suppose that the sector utility function is:

$$u(i) = \alpha(i) \log(c(i) - \gamma(i))$$

Here, γ is a sector-specific constant. If $\gamma(i)$ is positive then there is a consumption requirement, and if negative then the good may not be consumed at low income levels. To compute $\kappa(i)$:

$$u'(i) = \frac{\alpha(i)}{c(i)-\gamma(i)} \implies u''(i) = -\frac{\alpha(i)}{(c(i)-\gamma(i))^2} \implies \kappa(i) = \frac{\gamma(i)}{c(i)} - 1$$

In this case, the $\kappa(i)$ term changes as consumption within the sector increases or decreases. Therefore, in Proposition 1 both the expenditure terms and the $\kappa(i)$ terms themselves may change with trade costs.

is that the household has a two tiered utility function, and that the final good producer's problem is only the inner tier. This is equivalent to our formulation, and our terminology should not be seen to exclude this interpretation. We would like to thank an anonymous referee for pointing this to us.

4.3 Hyperbolic Absolute Risk Aversion

A very general preference specification, which encapsulates many commonly used functions is the Hyperbolic Absolute Risk Aversion—often used to analyse problems involving uncertainty—and we consider this case in our formulation. The sector utility function is:

$$u(i) = (\alpha(i)c(i) + \gamma(i))^{\sigma(i)}$$

Then,

$$u'(i) = \frac{\sigma(i)u(i)}{c(i) + \frac{\gamma(i)}{\alpha(i)}} \implies u''(i) = \frac{(\sigma(i) - 1)u'(i)}{c(i) + \frac{\gamma(i)}{\alpha(i)}} \implies \kappa(i) = \frac{1 + \frac{\gamma(i)}{\alpha(i)c(i)}}{\sigma(i) - 1}$$

As in the previous example, we see that the value of κ now varies with consumption, and that it can be increasing or decreasing depending on the values of parameters within the utility function.

4.4 CES Environments

A number of existing CES environments fit into our framework. Moreover, we allow for the parameters of those production environments to vary by sector. As in ACR, the models in Melitz (2003) (with Pareto-distributed productivity), Eaton and Kortum (2002), and Krugman (1980) all exhibit constant trade elasticities. Since our framework generalizes ACR, these same results go through for us. However, in our framework we can consider cases where, for example, every sector produces as in the Melitz model, but where every sector has a different tail parameter on the Pareto distribution from which productivity is drawn. An example of this type is considered in the quantitative example where production is Eaton and Kortum, but where the comparative advantage parameter varies by sector.

4.5 Variable Elasticity of Substitution

Now we provide an example to show how our approach can be implemented with a non-CES aggregator. As an example of a production structure that is not based on a CES aggregator, but that takes CES as a special case, consider the following constant returns to scale aggregator in a two country economy:

$$F_i = \left(x_{1i}^{\frac{\sigma_i - 1}{\sigma_i}} + x_{0i}^{\frac{\alpha_i - 1}{\alpha_i}} x_{1i}^{\left(\frac{\sigma_i - 1}{\sigma_i} - \frac{\alpha_i - 1}{\alpha_i}\right)} \right)^{\frac{\sigma_i}{\sigma_i - 1}}$$

Notice that the elasticity of substitution is now variable. That is, the elasticity of substitution is equal to:

$$\frac{d \log\left(\frac{F_{1i}}{F_{0i}}\right)}{d \log\left(\frac{x_{1i}}{x_{0i}}\right)} = -\alpha_i \frac{(\sigma_i - 1) x_{1i}^{1-1/\alpha_i} + (\sigma_i - \alpha_i) x_{0i}^{1-1/\alpha_i}}{(\sigma_i - 1) x_{1i}^{1-1/\alpha_i} + \frac{\sigma_i - \alpha_i}{\alpha_i} x_{0i}^{1-1/\alpha_i}}$$

In the CES case, where $\alpha_i = \sigma_i$, the elasticity of substitution is a constant equal to $-\alpha_i$ as expected.

Each good is produced with a linear technology in each country by competitive firms. Then the prices of these intermediate goods are:

$$q_n(i) = \frac{w_n}{z_n(i)}$$

where $z_n(i)$ is the productivity in sector i in country n. Solving the intermediary's problem above implies:

$$\begin{aligned} \frac{\partial \log(\lambda_{00}(i))}{\partial \log(\tau_{01})} \quad &= \quad \left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}\right) \lambda_{01}(i) \frac{\alpha_i - \sigma_i + \alpha_i(\sigma_i - 1)\lambda_{01}(i)}{\sigma_i - \alpha_i \lambda_{00}(i)} \\ \implies \quad & \rho(i) = \frac{\alpha(i) - \sigma(i) + \alpha(i)(\sigma(i) - 1)\lambda_{01}(i)}{\sigma(i) - \alpha(i) + \alpha(i)\lambda_{01}(i)} \end{aligned}$$

This shows that the sector-level trade elasticity varies as import penetration varies in the sector. This in itself is not surprising, as we would expect this to be true in any non-CES model. However, in this example we are able to compute the trade elasticity function exactly so we can see how it behaves as the sector approaches autarky. In this case, trade openness and the two elasticity terms α_i and σ_i are all that is needed to compute this. However, the sector-level elasticity may depend on other things as demonstrated in the next example.

4.6 Heterogeneous Exporters with Exponential Firm Size

Now suppose that production in sector i is done by monopolistically competitive firms with heterogeneous productivities. Unlike the more standard assumption that productivities are drawn from a Pareto distribution, as in Chaney (2008), we assume they are drawn from a gamma distribution. Final goods producers use a CES aggregator to combine intermediate goods from two identical countries into final output. That is:

$$Y(i) = \left(\int_{S_0} x_d(i,j)^{1-1/\sigma(i)} dj + \int_{S_1} x_m(i,j)^{1-1/\sigma(i)} dj \right)^{\frac{\sigma(i)}{\sigma(i)-1}}$$

where $Y(i)$ is the quantity of the final good produced in sector i, S_n is the set of operating intermediate goods producers in country n. There are an infinite number of potential firms in each country that make two discrete decisions. First, they decide whether or not to pay a fixed cost of f_e units of labor to draw a productivity from a distribution and operate domestically. Second, they decide whether or not to pay another fixed cost, f_x to enter the export market. These two discrete decisions pin down the measure of operating firms, and a productivity cut-off above which all firms enter the export market. Notice that aggregate profits net of entry fixed costs are zero.

To solve for the sector-level trade elasticity, first note that:

$$\frac{\lambda_{01}(i)}{\lambda_{00}(i)} = \tau_{01}^{1-\sigma(i)} \frac{\int_{z_m(i)}^{\infty} z^{\sigma(i)-1} g(z) dz}{\int_0^{\infty} z^{\sigma(i)-1} g(z) dz}$$

Here, $z_m(i)$ is the productivity cut-off that determines export status, and g is the density of productivity draws. Using the Fundamental Theorem of Calculus:

$$\frac{\partial \log \left(\frac{\lambda_{01}(i)}{\lambda_{00}(i)} \right)}{\log(\tau_{01})} = 1 - \sigma(i) - \frac{\partial z_m(i)}{\partial \log(\tau_{01})} \frac{z_m(i)^{\sigma(i)-1} g(z_m(i))}{\int_{z_m(i)}^{\infty} z^{\sigma(i)-1} g(z) dz}$$

The cutoff $z_m(i)$ changes with trade costs directly because of τ_{01} and indirectly from changes in sector price level $p(i)$. Using Lemma 3 for changes in $p(i)$ implies:

$$\frac{\partial z_m(i)}{\partial \log(\tau_{01})} = z_m(i) \frac{\sigma(i)}{\sigma(i)-1} \lambda_{00}(i)$$

Therefore:

$$\rho(i) = 1 - \sigma(i) - \frac{\sigma(i)}{\sigma(i)-1} \lambda_{00}(i) \frac{z_m^{\sigma(i)}(i) g(z_m(i))}{\int_{z_m(i)}^{\infty} z^{\sigma(i)-1} g(z) dz}$$

Up to this point, we have not needed to assume a functional form for g, which we do now. Suppose firms that pay the entry fixed cost draw a productivity from a gamma distribution with probability density function:

$$g(z) = \frac{\beta(i)^{\alpha(i)}}{\Gamma(\alpha(i))} z^{\alpha(i)-1} e^{-\beta(i)z}$$

where Γ is the gamma function. Furthermore, suppose that $\alpha(i) = 2 - \sigma(i)$, so that the firm size distribution is exponential.[6]

Now to measure the $\rho(i)$ terms it is necessary to know the fraction of firms in sector i that export, which we denote $\eta(i)$. This is because the extensive margin effect from changes in trade costs no longer have the constant marginal effect present with a Pareto distribution, as in Chaney (2008).

In this case, letting G be the cumulative density function of productivity draws we can write $\rho(i)$ as:

$$\rho(i) = 1 - \sigma(i) - \frac{\beta(i)\sigma(i)}{\sigma(i) - 1} \lambda_{00}(i) z_m(i)$$

$$\implies \rho(i) = 1 - \sigma(i) - \frac{\beta(i)\sigma(i)}{\sigma(i) - 1} (1 - \lambda_{01}(i)) G^{-1}(1 - \eta(i))$$

Therefore, we see that the production elasticity here is not a constant, and depends on import penetration $\lambda_{01}(i)$ as in the previous example, but now it also depends on the fraction of firms that export $\eta(i)$, both of which vary with trade costs. Therefore, in this model if one wished to compute the sector-level trade elasticity, one would need the sectoral elasticity of substitution $\sigma(i)$, the tail parameter $\beta(i)$, import penetration $\lambda_{01}(i)$ and the fraction of exporting firms $\eta(i)$. So, for example, suppose that two sectors had the same values of all these parameters and the same import penetration, if they had different fractions of firms exporting, then they would have different sector-level trade elasticities.

Some discussion is warranted about how the model is parametrized. As in ACR, this example demonstrates that not all parameters need to be known in order to compute the relevant trade elasticity. For example, we do not need to know the value of the fixed cost to export f_x to compute $\rho(i)$. However, that is not to say that all values of f_x imply the same $\rho(i)$. If one were to change f_x

[6] By "firm size" here we specifically refer to labor use, but the same statements are true of revenue. Formally we mean that, substituting the firm index for its productivity level:

$$l(i,z)g(z) \propto e^{-\beta(i)z} \quad \text{and} \quad j \in \{d, m\},\ q_j(i,z)x_j(i,z)g(z) \propto e^{-\beta(i)z}$$

and leave all other parameters unchanged, that would change $\eta(i)$ and $\lambda_{01}(i)$, which would change $\rho(i)$. Instead, ours is a sufficient statistics approach. All parametrizations that imply the same values of the observables $\eta(i)$ and $\lambda_{01}(i)$ imply the same value of $\rho(i)$.

5 Welfare Gains Away from Observed Levels of Trade

The results developed in Section 3 are useful for measuring the gains from small changes in trade costs because they show how to solve for the trade and welfare elasticities at observed levels of trade. However, the non-linearity inherent in non-CES models implies that the trade and welfare elasticities at observed levels of trade are different from the trade and welfare elasticities away from that point. Therefore, to measure the gains from large changes in trade costs we need to know how the trade elasticity changes.

First consider the gains from trade implied by a given trade elasticity, ε_T. Rearranging the definition of the welfare elasticity, equation (11), and applying Proposition 2 we get that:

$$\int_{\tau_{01}}^{\tau_{AUT}} d\log(W) = \int_{\tau_{01}}^{\tau_{AUT}} \frac{1}{\varepsilon_T(\tau)} d\log(\lambda_{00}) \tag{12}$$

If the trade elasticity is constant, the original formula from ACR follows immediately from equation (12). However, in the general case the trade elasticity is a function. In order to solve equation (12) we proceed by integrating by parts:

$$\log\left(\frac{W}{W^{AUT}}\right) = \frac{1}{\varepsilon_T(\tau_{01})}\log(\lambda_{00}) + \int_{\tau_{01}}^{\tau_{AUT}} -\frac{\log(\lambda_{00})}{\varepsilon_T(\tau)^2}\frac{\partial\varepsilon_T}{\partial\log(\tau)} d\log(\tau) \tag{13}$$

The first term of equation (13) is exactly the formula for the gains from trade from ACR; the second term depends on how the derivative of the trade elasticity on trade costs. Comparing the welfare gains from equation (13) to those from ACR is straightforward: consider two versions of the model outlined in Section 2 that, for a given level of iceberg costs τ_{01}, imply the same allocations. Suppose the first model is a non-CES model and the second one is a CES model. The non-CES model has a variable trade elasticity, $\varepsilon_{T,1}(\tau) < 0$, and the CES model has a constant trade elasticity, which we assume coincides with the trade elasticity of the non-CES model at the observed level of trade, $\varepsilon_{T,2}(\tau) = \varepsilon = \varepsilon_{T,1}(\tau_{01})$. In

Proposition 3 we establish conditions under which the welfare gains from trade in the non-CES model are always larger than in the CSE model and vice-versa.

Proposition 3 *The non-CES model has higher (lower) gains from trade at* τ_{01} *than the CES model if* $\varepsilon_{T,1}$ *is a monotonically increasing (decreasing) function of trade costs.*

The proof of Proposition 3 is in Appendix E. Recall that in Proposition 1 we established the formula for the trade elasticity. In Proposition 3 we show that the derivative of the trade elasticity with respect to trade costs is key to determine whether a particular non-CES model has larger or smaller gains from trade than a CES model. Hence, for any non-CES model, we can clearly establish the bias in the gains from trade as long as the derivative is monotone.

For instance, consider example of Section 4.5: variable elasticity of substitution. In it, the only relevant object for the trade elasticity is the sector-level trade elasticity,

$$\rho(i) = \frac{\alpha(i) - \sigma(i) + \alpha(i)(\sigma(i) - 1)\lambda_{01}(i)}{\sigma(i) - \alpha(i) + \alpha(i)\lambda_{01}(i)}$$

$$\implies \frac{\partial \rho(i)}{\partial \lambda_{01}(i)} = \frac{\alpha(i)\sigma(i)(\sigma(i) - \alpha(i))}{(\sigma(i) - \alpha(i) + \alpha(i)\lambda_{01}(i))^2}$$

Since $\rho(i)$ is monotone, Proposition 3 can be applied immediately. Whether $\rho(i)$ is increasing or decreasing with trade costs depends on the relative values of the sector-level parameters.

Similarly, consider example of Section 4.6: heterogeneous exporters with exponential firm size. There, the sector-level trade elasticity is:

$$\rho(i) = 1 - \sigma(i) - \frac{\beta(i)\sigma(i)}{\sigma(i) - 1}\lambda_{00}(i)z_m(i)$$

Recall that $z_m(i)$ is the export cut-off. Since the support of productivity draws is unbounded, clearly the export cut-off is increasing without bound as the country approaches autarky, and the fraction of expenditure on home production $\lambda_{00}(i)$ is increasing toward 1. Therefore, $\rho(i)$ is unambiguously decreasing as τ_{01} increases. In fact, in this example the marginal gains from trade near autarky are zero since the trade elasticity falls without bound.

These examples demonstrate that, given that the trade elasticity can be solved in closed form via Proposition 1 it is not difficult to determine how the trade elasticity changes with trade costs. As Proposition 3 shows, this in turn gives us information about how gains from trade differ away from observed levels of trade.

6 Application to a Quantitative Model

We now provide an example on how our results apply to a framework that already has non-CES parameters estimated. We choose to work with the framework of Caron, Fally and Markusen (2014), which we hereafter refer to as CFM, because it is consistent with many patterns of trade that are not matched by CES models, and cannot be solved in closed form.[7]

The household in CFM has constant relative income elasticity preferences over goods from many sectors, making sectors have different income elasticities.[8] Specifically, the utility function in country n is:

$$U_n = \sum_{i=1}^{50} \alpha_n(i) c_n(i)^{1-1/\sigma(i)}$$

In this case, the household trade elasticity for sector i is equal to $\kappa(i) = -\sigma(i)$.

Production in CFM is as in Eaton and Kortum (2002), but each sector is allowed to have a different shape parameter on the distribution of productivity draws, generating different sector-level trade elasticities. In each sector there is a unit mass of intermediate goods, each produced competitively in each country. In each country n and sector i, all producers of good j have the same productivity $z_n(i,j)$, which are drawn from a Frechet distribution within country-sectors pairs. That is, the density of intermediate goods in country n and sector i with productivity z is:

$$f(z) = e^{-T_n z^{-\theta(i)}}$$

The scale parameter T_n determines average productivity in country n, and the shape parameter $\theta(i)$ determines comparative advantage across countries. Note that this set-up does not allow for the scale parameter to vary across sectors within a country. In the model, labor markets are cleared by equilibrium wages in each country. The sector-level trade elasticity in sector i is $\rho(i) = -\theta(i)$, which is the sectoral comparative advantage parameter. The fact that different

[7]CFM presents several specifications. Here we are using the "theta-driven" model, which has sector heterogeneity in comparative advantage and a single factor of production.

[8]These preferences are the same as in example of Section 4.1.

sectors are allowed to have different values of $\theta(i)$ means that, when trade costs increase, some sectors reduce their sector level import penetration by more than others.

Therefore, the trade elasticity in this model is:

$$
\begin{aligned}
\varepsilon_T = \quad & \sum_{i=1}^{50}(-\theta(i)+\sigma(i)-1)\frac{\frac{X_{01}(i)}{X_{01}}\frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}} \\
& -\sum_{i=1}^{50}(\sigma(i)-1)\frac{X_{01}(i)}{X_{01}}\left[\frac{\sum_j \sigma(j)\frac{X_{00}(j)}{X_{00}}}{\sum_j \sigma(j)\frac{X_0(j)}{I_0}}\right]
\end{aligned} \tag{14}
$$

Equation (14) has two types of terms. First, there are the elasticity terms $\theta(i)$ and $\sigma(i)$. We use direct estimates from CFM for these values.[9] Second, there are expenditure share terms. Following CFM we use the GTAP data set from Aguiar, McDougall and Narayanan (2012), which includes production and trade data by sector in a large number of countries.[10]

One problem with applying the elasticity estimates from CFM is that their empirical strategy only identifies the set of $\sigma(i)$ and $\theta(i)$ terms up to multiplicative constants for each group of elasticity estimates. Therefore, we are left with two degrees of freedom in choosing the averages of these vectors. We choose two targets. First, we match the average value of $\theta(i)$ across sectors to 4 based on Simonovska and Waugh (2014), which identifies the shape parameter in the Eaton and Kortum (2002) model correcting for finite sample bias.[11] Second, using all data and the estimates of $\theta(i)$, we choose the average of $\sigma(i)$ to set the average country-level aggregate trade elasticity to -7.5, which is the midpoint of the range of trade elasticity estimates from Anderson and Wincoop (2004). We choose this for comparability to the existing literature that uses this range of aggregate trade elasticity estimates, such as ACR.

While these choices certainly affect our estimates of the average marginal gains from trade, our goal is to see how this generalized framework affects two things that these choices do not affect. First, we show that this non-CES environment implies substantial variation in trade elasticities across countries, which is impossible in CES models. Second, we show that the non-CES environment

[9]Special thanks to Justin Caron for providing us with these elasticity estimates.

[10]From GTAP 8, we make use of data from 118 countries (seen Table 1) and following CFM, we remove seven sectors composed of raw materials or those that are not traded, leaving fifty sectors spanning agriculture, manufacturing and services. We use data on country-sector imports, and country-sector final expenditure.

[11]Our choice of 4 is close to the midpoint of the range they estimate, which is 2.79 to 4.46.

has substantially different gains from trade away from observed levels of trade than does a CES model with the same aggregate trade elasticity.

6.1 Measuring the Trade and Welfare Elasticities

The trade elasticity is computed using the formula in equation (14). The results are presented in the first column of Table 1. Aggregate trade elasticities range from -4.42 (Peru) to -11.37 (Luxembourg) with a standard deviation of 1.41 and average of -7.5. By Proposition 2, the welfare elasticity is just the reciprocal of these trade elasticities. Therefore the welfare elasticities range from -0.09 to -0.23, meaning that a marginal change in import penetration has a 2.57 greater effect on real income in Peru than in Luxembourg. In a CES model, the ACR result implies that there is no difference across countries.

There is a strong correlation between import penetration and the trade elasticity, which is shown in Figure 1. The correlation between these two is -0.60. To show why this is the case, we prove a simple limiting result.

Proposition 4 *Let* $\theta^{MIN} = \min\{\theta(i)\}$. *Then:*

$$\lim_{\tau_{01} \to \infty} \varepsilon_T = -\theta^{MIN}$$

The proof of Proposition 4 is available in the appendix. The interpretation of this result is that, as the country approaches autarky, country-level trade elasticities approach a known fixed number. In our parametrization, the limit is -2.21, as the industry with the lowest $\theta(i)$ is leather products. Therefore, the welfare elasticity near autarky is -0.45, which is more than twice the marginal effect of any country in the sample. This demonstrates that as countries go to autarky, their aggregate trade elasticities are higher than close to observed levels of trade, consistent with the pattern seen in the computed elasticities provided.

Even among countries of similar import penetration levels, there is considerable variation in trade elasticities. We provide a decomposition to understand this heterogeneity.

6.2 Decomposition of Trade Elasticity

Notice that the trade elasticity, equation (14), can be broken into two terms:

$$\Theta = -\sum_{i=1}^{50} \theta(i) \frac{\frac{X_{01}(i)}{X_{01}} \frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}} \quad (15)$$

$$\Sigma = \sum_{i=1}^{50} (\sigma(i) - 1) \frac{X_{01}(i)}{X_{01}} \left[\frac{\frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}} - \frac{\sum_j \sigma(j) \frac{X_{00}(j)}{X_{00}}}{\sum_j \sigma(j) \frac{X_0(j)}{I_0}} \right] \quad (16)$$

The Θ term includes only sector-level trade elasticities, and the Σ term includes only household elasticities. These are reported in Table 1.[12] Inspection of the table shows that the Σ term has much more variation across countries than the Θ term. Writing out the usual decomposition of variance:

$$\varepsilon_T = \Theta + \Sigma \implies Var(\varepsilon_T) = Var(\Theta) + Var(\Sigma) + 2Cov(\Theta, \Sigma)$$

The variance of trade elasticities is 1.99, the variance of Θ is 0.17, the variance of Σ is 3.15, and the covariance of Θ and Σ is -0.66. This shows that the demand elasticities are the major determinants of cross-country variation in aggregate trade elasticities.

To understand why there is so much variation in Σ we now consider two countries with very similar import penetration, but very different trade elasticities: Finland and Armenia. As shown in Table 1, Finland has import penetration of 17.03% and a trade elasticity of -5.66, while Armenia has import penetration of 16.59% and a trade elasticity of -9.23. From the results of the decomposition above listed on Table 1 we can again see that the difference in the Σ term is responsible for the large difference in trade elasticities, since Θ is actually higher (closer to zero) in Armenia than in Finland. What makes Σ so much lower in Finland is that the profile of domestic expenditure across sectors $X_{00}(i)$ closely follows the profile of total expenditures $X_0(i)$ whereas in Armenia there are several sectors in which spending is high, but domestic production is very low. This is depicted in Figure 2. To understand why this is important, in the formula for Σ above we see that:

$$\forall i, \frac{X_{00}(i)}{X_{00}} = \frac{X_0(i)}{I_0} \implies \Sigma = 0$$

[12] Notice that the weights appearing in the definition of Θ add to a number less than one whenever there is cross-sectoral variation in import penetration. Therefore, even though the average value of θ is 4, all countries have a Θ greater than -4.

The Σ term should be interpreted as a composition effect. As trade costs change, consumption is reallocated across sectors and in particular is reallocated away from high import penetration sectors, which experience the highest price increases. The Σ term measures how much trade is reduced as consumption in the heavy import sectors decreases. However, if all sectors have exactly the same import penetration, then, there can be no composition effect because no matter how consumption is reallocated, aggregate import penetration remains the same. Therefore, in Finland, the fact that there is low variation in import penetration across sectors implies that Σ is much closer to zero than in Armenia.

6.3 Comparison to Autarky

Now we measure changes in real income moving from autarky to observed levels of trade and compare the increase in real income to the increase in income implied by CES models. In Section 5 we show that the main determinant of this is the derivative of the trade elasticity with respect to trade costs. As Proposition 5 demonstrates, every country's trade elasticity near autarky is known, and is higher than any country's trade elasticity at observed levels of trade. If this relationship is monotone, then Proposition 4 says that gains from trade are higher in this model than in CES models.

To measure the size of this effect we compute the real income needed to make each country indifferent between observed levels of trade and autarky.[13] To solve for the equilibrium in autarky many more parameters are needed than in the previous calculation, as equilibrium wages must be calculated. In particular, we need the vector of preference weights $\alpha(i)$, sector-level trade costs $\tau(i)$, absolute advantage parameter T_n and population size L_0. We select these parameters for each country to exactly match sector-level imports, sector-level production, population as a fraction of world population, and GDP as a fraction of world GDP.[14] Given these parameters, we then solve for the compensating variation that equates utility in the observed trade equilibrium and autarky equilibrium.

The results of this exercise are shown in the True Gains column of Table 1. For comparison, the ACR Gains columns list the gains from trade implied by the ACR formula under two different assumptions. In ACR Gains (constant), gains

[13]Since there is no profit and the domestic wage is numeraire, we are computing how many more efficiency labor units the country would have to be endowed with to be indifferent between autarky and observed trade.

[14]World population and world GDP are defined respectively as the sum of population and sum of GDP in the 118 countries in our sample.

from trade are computed assuming that the trade elasticity is constant and equal to -7.5 in each country, which is the average trade elasticity across countries. In ACR Gains (marginal), gains are computed using the ACR formula and the country-specific aggregate trade elasticity. Propositions 4 and 5 imply that the True Gains entries must be higher than the ACR Gains (marginal) entries for each country. In thirteen countries the ACR Gains (constant) entry is higher than the true gains. On average in those countries, true gains are 6.4% lower. These countries are small and highly open to trade. In total they account for 1.4% of world population and have an average import penetration of 33.3%.

The population-weighted average difference between the actual gains from trade and those implied by the ACR formula is 60%, and the GDP-weighted average is 59%. As before, those countries with the lowest import penetration have the largest disagreement between the ACR prediction and the actual gains. This is because of the non-linear effect that trade has on real income. The majority of gains are realized near autarky, so countries with higher import penetration realize a diminishing welfare effect.

7 Conclusion

In this paper we provide a method for measuring gains from trade in non-CES trade models. We show that non-CES models have interesting properties that CES models lack, such as a role for patterns of production and trade in determining the welfare gains from trade. In a quantitative application, we show that these mechanisms are important. One way to view our contribution is that we demonstrate how to apply the ACR framework of measuring welfare gains from trade to models that are non-CES. Future work extending the results in this paper to allow for heterogeneous income individuals, in line with Fajgelbaum and Khandelwal (forthcoming), is a good avenue for future research. Similarly, more work extending the results of ACR, relaxing some of their assumptions seems a fruitful avenue for future research—in Brooks and Pujolas (2016) we extend the results of ACR to dynamic model of international trade, relaxing the assumption of the model being static.

References

Adao, Rodrigo, Arnaud Costinot, and Dave Donaldson. 2015. "Non-parametric Counterfactual Predictions in Neoclassical Models of International Trade." Working Paper.

Aguiar, Angel, Robert McDougall, and Badri Narayanan. 2012. "Global Trade, Assistance, and Production: The GTAP 8 Data Base." *https://www.gtap.agecon.purdue.edu/databases/v7/v7_doco.asp.*

Allen, Treb, Costas Arkolakis, and Yuta Takahashi. 2014. "Universal Gravity." Working Paper.

Anderson, James. 1979. "A Theoretical Foudnation for the Gravity Equation." *American Economic Review*, 69(1): 106–116.

Anderson, James, and Eric Van Wincoop. 2004. "Trade Costs." *Journal of Economic Literature*, 42(3): 691–751.

Ardelean, Adina, and Volodymyr Lugoskyy. 2010. "Domestic Productivity and Variety Gains from Trade." *Journal of International Economics*, 80(2): 280–291.

Arkolakis, Costas, Arnaud Costinot, and Andres Rodriguez-Clare. 2012. "New Trade Models, Same Old Gains?" *American Economic Review*, 102(1): 94–130.

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson, and Andres Rodriguez-Clare. 2014. "The Elusive Pro-Competitive Effects of Trade." Working Paper.

Bergstrand, Jeffrey. 1985. "The Gravity Equation in International Trade: Some Microeconomic Foundations and Empirical Evidence." *Review of Economics and Statistics*, 67(3): 474–481.

Blonigen, Bruce, and Anson Soderbery. 2010. "Measuring the Benefits of Foreign Product Variety with an Accurate Variety Set." *Journal of International Economics*, 82(2): 168–180.

Broda, Christian, and David Weinstein. 2006. "Globalization and Gains from Variety." *Quarterly Journal of Economics*, 121(2): 541–585.

Brooks, Wyatt, and Pau Pujolas. 2016. "Capital Accumulation and the Welfare Gains from Trade." Working Paper.

Caron, Justin, Thibault Fally, and James Markusen. 2014. "International Trade Puzzles: A Solution Linking Production and Preferences." *Quarterly Journal of Economics*, 121(2): 541–585.

Chaney, Thomas. 2008. "Distorted Gravity: the Intensive and Extensive Margins of International Trade." *American Economic Review*, 98(4): 1707–1721.

Cooke, Richard. 2014. *Infinite Matrices and Sequence Spaces.* Dover Publications.

Eaton, Jonathon, and Samuel Kortum. 2002. "Technology, Geography and Trade." *Econometrica*, 70(5): 1741–1779.

Edmond, Chris, Virgiliu Midrigan, and Daniel Xu. 2015. "Competition, Markups and the Gains from International Trade." *American Economic Review*, 105(10): 3183–3221.

Fajgelbaum, Pablo, and Amit Khandelwal. forthcoming. "Measuring the Unequal Gains from Trade." *Quarterly Journal of Economics.*

Feenstra, Robert. 1994. "New Product Varieties and the Measurement of International Prices." *American Economic Review*, 84(1): 157–177.

Feenstra, Robert. 2014. "Restoring the Product Variety and Pro-competitive Gains from Trade with Heterogeneous Firms and Bounded Productivity." NBER Working Paper No. 19833.

Feenstra, Robert, and David Weinstein. 2010. "Globalization, Markups and U.S. Welfare." NBER Working Paper No. 15749.

Fieler, Ana. 2011. "Non-Homotheticity and Bilateral Trade: Evidence and a Quantitative Explanation." *Econometrica*, 79(4): 1069–1101.

Holmes, Thomas, Wen-Tai Hsu, and Sanghoon Lee. 2014. "Allocative Efficiency, Mark-ups and the Welfare Gains from Trade." *Journal of International Economics*, 94(2): 195–206.

Krugman, Paul. 1980. "Scale Economies, Product Differentiation and the Pattern of Trade." *American Economic Review*, 70(5): 950–959.

Markusen, James. 1986. "Explaining the Volume of Trade: An Ecletic Approach." *American Economic Review*, 76(5): 1002–1011.

Melitz, Marc. 2003. "The Impact of Trade on Aggregate Industry Productivity and Intra-Industry Reallocation." *Econometrica*, 71(6): 1695–1725.

Melitz, Marc, and Stephen Redding. 2014. "Missing Gains from Trade?" *American Economic Review*, 104(5): 317–321.

Melitz, Marc, and Stephen Redding. 2015. "New Trade Models, New Welfare Implications." *American Economic Review*, 105(3): 1105–1146.

Novy, Dennis. 2013. "International Trade Without CES: Estimating Translog Gravity." *Journal of International Economics*, 89(2): 271–282.

Ossa, Ralph. 2015. "Why Trade Matters After All." *Journal of International Economics*, 97(2): 266–277.

Simonovska, Ina. 2015. "Income Differences and Prices of Tradables: Insights from an Online Retailer." *Review of Economic Studies*, 82(4): 1612–1656.

Simonovska, Ina, and Michael Waugh. 2014. "The Elasticity of Trade: Estimates and Evidence." *Journal of International Economics*, 92(1): 34–50.

Soderbery, Anson. 2015. "Estimating Import Supply and Demand Elasticities: Analysis and Implications." *Journal of International Economics*, 96(1): 1–17.

Figure 1: Trade Elasticity by Import Penetration

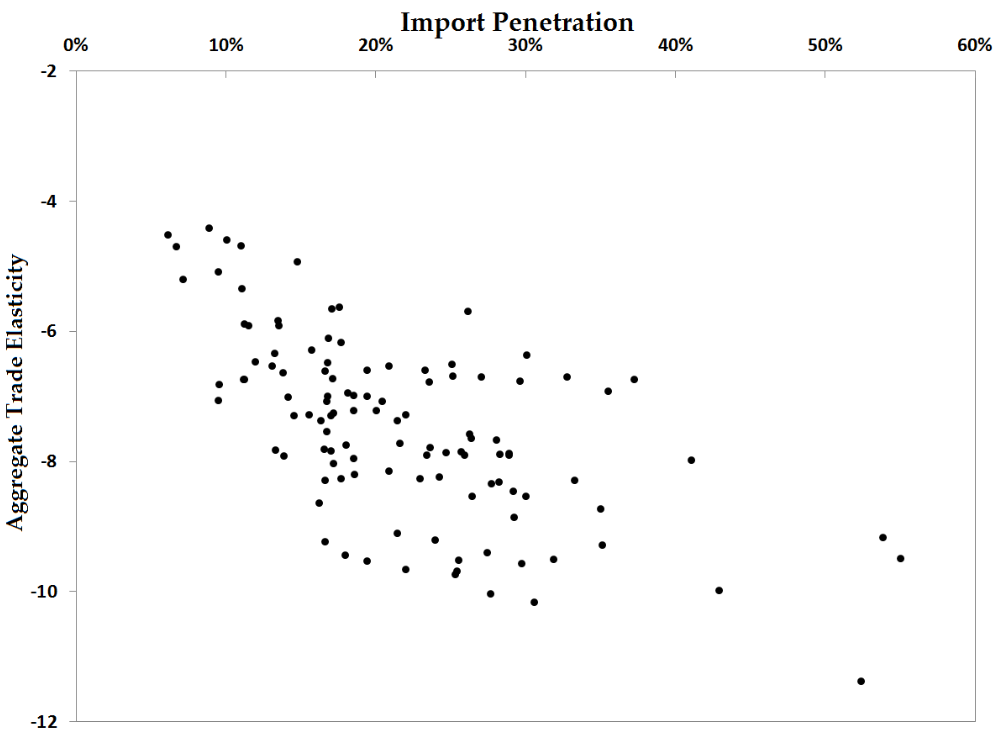
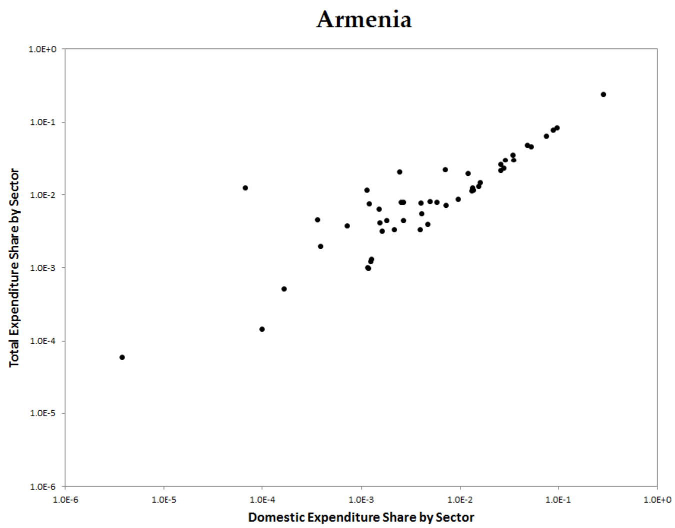


Figure 2: Comparing Domestic and Total Expenditure Shares, Armenia and Finland



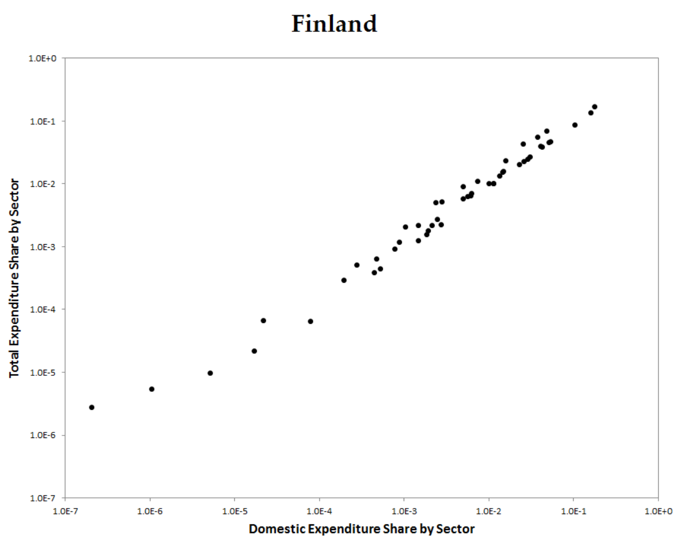


Table 1: Trade Elasticity and Decomposition

Country	Import Penetration	Trade Elasticity	Production Term (Θ)	Preference Term (Σ)	Country	Import Penetration	Trade Elasticity	Production Term (Θ)	Preference Term (Σ)
Australia	9.52%	-6.81	-2.25	-4.56	Poland	16.62%	-6.60	-2.20	-4.40
New Zealand	11.24%	-5.88	-2.50	-3.38	Portugal	15.69%	-6.28	-2.39	-3.89
China	10.03%	-4.59	-2.51	-2.08	Slovakia	23.61%	-7.78	-2.01	-5.77
Hong Kong	26.44%	-8.53	-1.97	-6.57	Slovenia	26.37%	-7.64	-2.01	-5.63
Japan	6.66%	-4.70	-2.70	-1.99	Spain	13.47%	-5.83	-2.53	-3.30
South Korea	14.78%	-4.93	-2.63	-2.30	Sweden	17.66%	-6.17	-2.47	-3.69
Mongolia	30.56%	-10.17	-1.33	-8.83	United Kingdom	13.27%	-6.33	-2.52	-3.81
Taiwan	25.10%	-6.51	-2.17	-4.34	Switzerland	25.30%	-9.73	-1.68	-8.05
Cambodia	33.25%	-8.29	-1.40	-6.89	Norway	16.76%	-6.48	-2.49	-3.99
Indonesia	13.52%	-5.91	-2.43	-3.48	Iceland	21.45%	-7.37	-2.22	-5.15
Laos	17.98%	-9.44	-1.27	-8.16	Albania	28.26%	-7.89	-2.32	-5.57
Malaysia	29.60%	-6.76	-2.11	-4.65	Bulgaria	24.70%	-7.87	-2.11	-5.76
Philippines	22.02%	-7.28	-1.93	-5.35	Belarus	18.59%	-8.20	-1.75	-6.45
Singapore	35.51%	-6.91	-2.45	-4.47	Croatia	19.44%	-7.00	-2.21	-4.79
Thailand	25.12%	-6.69	-2.18	-4.51	Romania	16.77%	-6.99	-2.06	-4.93
Vietnam	41.05%	-7.98	-1.57	-6.40	Russia	11.93%	-6.47	-2.36	-4.11
Bangladesh	13.82%	-6.64	-1.97	-4.67	Ukraine	18.51%	-7.95	-2.00	-5.94
India	9.50%	-5.09	-2.63	-2.46	Moldova	34.99%	-8.72	-1.91	-6.82
Nepal	16.98%	-7.84	-1.85	-5.99	Kazakhstan	17.02%	-7.29	-2.35	-4.95
Pakistan	13.33%	-7.82	-1.93	-5.90	Kyrgyzstan	27.42%	-9.40	-1.37	-8.03
Sri Lanka	20.43%	-7.07	-1.90	-5.16	Armenia	16.59%	-9.23	-1.72	-7.52
Canada	14.17%	-7.01	-2.25	-4.76	Azerbaijan	25.54%	-9.52	-2.01	-7.51
USA	7.16%	-5.20	-2.56	-2.64	Georgia	23.97%	-9.20	-1.45	-7.75
Mexico	13.84%	-7.91	-1.85	-6.06	Bahrain	27.63%	-10.04	-1.43	-8.61
Argentina	11.19%	-6.73	-2.22	-4.51	Iran	18.11%	-6.94	-2.29	-4.65
Bolivia	16.19%	-8.63	-1.84	-6.79	Israel	18.54%	-7.21	-2.24	-4.97
Brazil	6.15%	-4.51	-2.77	-1.75	Kuwait	25.69%	-7.85	-2.31	-5.54
Chile	14.55%	-7.29	-2.13	-5.16	Oman	29.74%	-9.57	-1.65	-7.92
Colombia	9.47%	-7.06	-1.96	-5.10	Qatar	20.05%	-7.22	-2.06	-5.16
Ecuador	16.32%	-7.37	-2.09	-5.28	Saudi Arabia	30.10%	-6.36	-2.81	-3.55
Paraguay	21.41%	-9.10	-1.39	-7.71	Turkey	13.09%	-6.53	-2.12	-4.40
Peru	8.86%	-4.42	-2.70	-1.72	UAE	35.09%	-9.28	-1.57	-7.70
Uruguay	16.55%	-7.81	-2.04	-5.77	Egypt	19.39%	-6.59	-2.31	-4.28
Venezuela	11.20%	-6.73	-2.17	-4.57	Morocco	17.55%	-5.62	-2.44	-3.19
Costa Rica	24.24%	-8.24	-1.78	-6.46	Tunisia	28.88%	-7.90	-1.84	-6.07
Guatemala	20.86%	-8.15	-1.66	-6.49	Algeria	16.70%	-7.54	-2.00	-5.54
Honduras	29.17%	-8.46	-1.44	-7.02	Benin	42.94%	-9.97	-0.87	-9.10
Nicaragua	28.23%	-8.31	-1.84	-6.47	Burkina Faso	17.15%	-7.26	-1.89	-5.36
Panama	25.43%	-9.68	-1.31	-8.37	Cameroon	11.08%	-5.34	-2.62	-2.72
El Salvador	20.87%	-6.54	-2.22	-4.32	Cote d'Ivoire	17.13%	-6.72	-2.16	-4.56
Belize	26.13%	-5.69	-2.55	-3.15	Ghana	21.61%	-7.72	-1.79	-5.93
Austria	23.41%	-7.90	-2.12	-5.78	Guinea	27.74%	-8.34	-1.68	-6.65
Belgium	31.88%	-9.50	-1.73	-7.77	Nigeria	22.97%	-8.26	-1.75	-6.51
Cyprus	28.89%	-7.88	-2.00	-5.87	Senegal	28.05%	-7.66	-1.93	-5.73
Czech Republic	23.30%	-6.60	-2.17	-4.43	Togo	55.03%	-9.48	-1.07	-8.41
Denmark	23.54%	-6.77	-2.63	-4.15	Ethiopia	17.67%	-8.27	-1.54	-6.73
Estonia	29.25%	-8.85	-1.73	-7.13	Kenya	15.56%	-7.28	-1.93	-5.34
Finland	17.03%	-5.66	-2.49	-3.17	Madagascar	17.18%	-8.02	-1.72	-6.30
France	11.52%	-5.91	-2.45	-3.47	Malawi	19.42%	-9.53	-1.40	-8.13
Germany	16.83%	-6.10	-2.49	-3.62	Mauritius	37.24%	-6.73	-2.19	-4.54
Greece	18.50%	-6.98	-2.21	-4.77	Mozambique	30.03%	-8.53	-1.66	-6.87
Hungary	27.02%	-6.70	-2.24	-4.46	Rwanda	18.01%	-7.75	-1.86	-5.88
Ireland	32.79%	-6.70	-2.80	-3.91	Tanzania	21.99%	-9.66	-1.27	-8.39
Italy	11.03%	-4.68	-2.82	-1.86	Uganda	16.61%	-8.29	-1.74	-6.55
Latvia	26.27%	-7.58	-2.06	-5.52	Zambia	13.50%	-9.44	-1.64	-7.80
Lithuania	25.90%	-7.90	-1.96	-5.94	Zimbabwe	38.07%	-7.81	-2.15	-5.66
Luxembourg	52.37%	-11.37	-1.96	-9.41	Botswana	23.39%	-9.91	-1.42	-8.49
Malta	53.83%	-9.16	-1.89	-7.27	Namibia	24.30%	-8.91	-1.59	-7.31
Netherlands	16.75%	-7.08	-2.43	-4.65	South Africa	11.21%	-5.52	-2.50	-3.02

Table 2: Gains from Trade: Autarky to Observed Trade

Country	Trade Elasticity	ACR Gains (constant)	ACR Gains (marginal)	True Gains	Country	Trade Elasticity	ACR Gains (constant)	ACR Gains (marginal)	True Gains
Australia	-6.81	1.34%	1.48%	1.83%	Poland	-6.60	2.45%	2.79%	3.34%
New Zealand	-5.88	1.60%	2.05%	2.45%	Portugal	-6.28	2.30%	2.75%	3.32%
China	-4.59	1.42%	2.33%	2.73%	Slovakia	-7.78	3.66%	3.52%	4.21%
Hong Kong	-8.53	4.18%	3.66%	4.62%	Slovenia	-7.64	4.17%	4.09%	4.95%
Japan	-4.70	0.92%	1.48%	1.76%	Spain	-5.83	1.95%	2.51%	2.97%
South Korea	-4.93	2.16%	3.30%	3.81%	Sweden	-6.17	2.62%	3.20%	3.87%
Mongolia	-10.17	4.98%	3.65%	4.44%	United Kingdom	-6.33	1.92%	2.27%	2.76%
Taiwan	-6.51	3.93%	4.54%	5.30%	Switzerland	-9.73	3.97%	3.04%	3.95%
Cambodia	-8.29	5.54%	5.00%	5.73%	Norway	-6.48	2.48%	2.87%	3.52%
Indonesia	-5.91	1.96%	2.49%	3.30%	Iceland	-7.37	3.27%	3.33%	3.97%
Laos	-9.44	2.68%	2.12%	2.68%	Albania	-7.89	4.53%	4.30%	5.26%
Malaysia	-6.76	4.79%	5.33%	5.99%	Bulgaria	-7.87	3.86%	3.67%	4.43%
Philippines	-7.28	3.37%	3.47%	4.40%	Belarus	-8.20	2.78%	2.54%	3.14%
Singapore	-6.91	6.02%	6.55%	7.65%	Croatia	-7.00	2.92%	3.14%	3.78%
Thailand	-6.69	3.93%	4.42%	5.16%	Romania	-6.99	2.48%	2.66%	3.23%
Vietnam	-7.98	7.30%	6.85%	7.97%	Russia	-6.47	1.71%	1.98%	2.41%
Bangladesh	-6.64	2.00%	2.26%	2.89%	Ukraine	-7.95	2.77%	2.61%	3.16%
India	-5.09	1.34%	1.98%	2.32%	Moldova	-8.72	5.91%	5.06%	5.86%
Nepal	-7.84	2.51%	2.40%	3.01%	Kazakhstan	-7.29	2.52%	2.59%	3.14%
Pakistan	-7.82	1.93%	1.85%	2.46%	Kyrgyzstan	-9.40	4.37%	3.47%	4.10%
Sri Lanka	-7.07	3.09%	3.29%	4.47%	Armenia	-9.23	2.45%	1.98%	2.51%
Canada	-7.01	2.06%	2.20%	2.68%	Azerbaijan	-9.52	4.01%	3.15%	3.95%
USA	-5.20	0.99%	1.44%	1.71%	Georgia	-9.20	3.72%	3.02%	3.75%
Mexico	-7.91	2.01%	1.90%	2.44%	Bahrain	-10.04	4.41%	3.27%	3.92%
Argentina	-6.73	1.59%	1.78%	2.20%	Iran	-6.94	2.70%	2.92%	3.54%
Bolivia	-8.63	2.38%	2.07%	2.69%	Israel	-7.21	2.77%	2.89%	3.57%
Brazil	-4.51	0.85%	1.41%	1.63%	Kuwait	-7.85	4.04%	3.85%	4.78%
Chile	-7.29	2.12%	2.18%	2.75%	Oman	-9.57	4.82%	3.76%	4.60%
Colombia	-7.06	1.34%	1.42%	1.91%	Qatar	-7.22	3.03%	3.15%	3.66%
Ecuador	-7.37	2.40%	2.45%	3.15%	Saudi Arabia	-6.36	4.89%	5.79%	6.64%
Paraguay	-9.10	3.27%	2.68%	3.37%	Turkey	-6.53	1.89%	2.17%	2.63%
Peru	-4.42	1.24%	2.12%	2.42%	UAE	-9.28	5.93%	4.77%	5.95%
Uruguay	-7.81	2.44%	2.34%	2.94%	Egypt	-6.59	2.92%	3.32%	4.22%
Venezuela	-6.73	1.60%	1.78%	2.43%	Morocco	-5.62	2.61%	3.49%	4.02%
Costa Rica	-8.24	3.77%	3.43%	4.50%	Tunisia	-7.90	4.65%	4.41%	5.30%
Guatemala	-8.15	3.17%	2.91%	3.78%	Algeria	-7.54	2.47%	2.45%	3.05%
Honduras	-8.46	4.71%	4.16%	5.03%	Benin	-9.97	7.77%	5.79%	9.07%
Nicaragua	-8.31	4.52%	4.07%	5.14%	Burkina Faso	-7.26	2.54%	2.63%	3.40%
Panama	-9.68	3.99%	3.08%	3.67%	Cameroon	-5.34	1.58%	2.22%	3.02%
El Salvador	-6.54	3.17%	3.64%	4.22%	Cote d'Ivoire	-6.72	2.54%	2.83%	4.16%
Belize	-5.69	4.12%	5.46%	6.20%	Ghana	-7.72	3.30%	3.20%	4.11%
Austria	-7.90	3.62%	3.43%	4.27%	Guinea	-8.34	4.43%	3.97%	5.16%
Belgium	-9.50	5.25%	4.12%	5.39%	Nigeria	-8.26	3.54%	3.21%	3.87%
Cyprus	-7.88	4.65%	4.42%	5.33%	Senegal	-7.66	4.49%	4.39%	6.37%
Czech Republic	-6.60	3.60%	4.10%	4.83%	Togo	-9.48	11.24%	8.79%	10.66%
Denmark	-6.77	3.64%	4.04%	4.83%	Ethiopia	-8.27	2.63%	2.38%	2.91%
Estonia	-8.85	4.72%	3.99%	4.85%	Kenya	-7.28	2.28%	2.35%	3.31%
Finland	-5.66	2.52%	3.36%	3.85%	Madagascar	-8.02	2.54%	2.38%	3.44%
France	-5.91	1.65%	2.09%	2.51%	Malawi	-9.53	2.92%	2.29%	3.22%
Germany	-6.10	2.49%	3.07%	3.73%	Mauritius	-6.73	6.41%	7.16%	8.92%
Greece	-6.98	2.77%	2.97%	3.56%	Mozambique	-8.53	4.88%	4.28%	6.01%
Hungary	-6.70	4.29%	4.82%	5.54%	Rwanda	-7.75	2.68%	2.60%	3.06%
Ireland	-6.70	5.44%	6.11%	6.63%	Tanzania	-9.66	3.37%	2.60%	3.29%
Italy	-4.68	1.57%	2.53%	2.87%	Uganda	-8.29	2.45%	2.22%	2.82%
Latvia	-7.58	4.15%	4.10%	4.87%	Zambia	-9.44	1.95%	1.55%	2.00%
Lithuania	-7.90	4.08%	3.87%	4.67%	Zimbabwe	-7.81	6.60%	6.33%	7.79%
Luxembourg	-11.37	10.40%	6.74%	8.62%	Botswana	-9.91	3.62%	2.73%	3.59%
Malta	-9.16	10.85%	8.80%	9.25%	Namibia	-8.91	3.78%	3.18%	3.82%
Netherlands	-7.08	2.47%	2.62%	3.37%	South Africa	-5.52	1.60%	2.18%	2.65%

A Appendices

Appendix A: Multiple Trading Partners

We now analyze the effects on country 0 of changing trade costs with its N other trading partners, which we index $n = 1, ..., N$. We assume that a parameter τ governs trade costs between country 0 and all its trading partners.[15] With more trading partners, we need to change the definitions of the elasticities. First, we define the trade elasticity as:

$$\varepsilon_T = \frac{\frac{\partial \log\left(\frac{1-\lambda_{00}}{\lambda_{00}}\right)}{\partial \log(\tau)}}{\sum_{n=1}^{N} \frac{\lambda_{0n}}{1-\lambda_{00}} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)}$$

The demand elasticities $\kappa(i)$ have the same definition as before, but the definition of the production elasticity is now:

$$\rho(i) = \frac{\frac{\partial \log\left(\frac{1-\lambda_{00}(i)}{\lambda_{00}(i)}\right)}{\partial \log(\tau)}}{\sum_{n=1}^{N} \frac{\lambda_{0n}(i)}{1-\lambda_{00}(i)} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)}$$

Notice that if all N trading partners are identical, then this trade elasticity is the same as the case of a single trading partner.

Proposition 5 *Whenever $\lambda_{00} \in (0, 1)$,*

$$\begin{aligned}\varepsilon_T = & \int_\Omega [\rho(i) - (1 + \kappa(i))] \frac{\sum_{n=1}^{N} \lambda_{0n}(i) \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)}{\sum_{n=1}^{N} \lambda_{0n} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)} \frac{X_{00}(i)}{X_{00}} di \\ & + \int_\Omega (1 + \kappa(i)) \frac{\sum_{n=1}^{N} \lambda_{0n}(i) \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)}{\sum_{n=1}^{N} \lambda_{0n} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right)} X_0(i) di \frac{\int_\Omega \kappa(i) \frac{X_{00}(i)}{X_{00}} di}{\int_\Omega \kappa(i) X_0(i) di}\end{aligned}$$

Proof *First, note that the analogue of Lemma 3 now is:*

$$\frac{\partial \log(p(i))}{\partial \log(\tau)} = \sum_{n=1}^{N} \left(1 + \frac{\partial \log(w_n)}{\partial \log(\tau)}\right) \lambda_{0n}(i)$$

In Appendix C we prove Proposition 1 of the main text. Notice that nothing in the proof presented in Appendix C for Proposition 1 is changed up to this

[15] That is, $\tau_{0j} = \tilde{\tau}_{0j} \tau$ and we will be considering changes in the common component τ.

equation:

$$\frac{\partial\log(\lambda_{00})}{\partial\log(\tau)} = \int_\Omega\left[\frac{\partial\log(\lambda_{00}(i))}{\partial\log(\tau)}+(1+\kappa(i))\frac{\partial\log(p_0(i))}{\partial\log(\tau)}\right]\frac{X_{00}(i)}{X_{00}}di - \int_\Omega\kappa(i)\frac{X_{00}(i)}{X_{00}}di\frac{\int_\Omega\frac{\partial\log(p_0(i))}{\partial\log(\tau)}(1+\kappa(i))X_0(i)di}{\int_\Omega\kappa(i)X_0(i)di}$$

Then the result follows immediately by substituting in the above equation for price changes and the definition of ε_T.

Appendix B: Non-Separable Preferences

For simplicity in the body of the paper we assumed that the utility function was additively seperable. In this section we dispense with that assumption and show that our results are unchanged. That is, the household's problem is now:

$$\max U\left(\{\{c_n(i)\}_{i\in\Omega_n}\}_{n=0}^N\right)$$
$$s.t. \quad : \quad \sum_{n=0}^N \tau_{0n}p_n(i)c_n(i)\le I_0$$

This specification allows for complementarity or substitutability of goods both within and between countries.

Let H be the Hessian matrix of U. Because U is strictly concave and twice continuously differentiable, H is negative definite and invertible.[16]Row i of H contains all the second order partial derivatives of good i with all other goods.

The analogous demand elasticity to what we had before is:

$$\beta(i,j)=H^{-1}_{(i,j)}\frac{\frac{\partial U}{\partial c(i)}}{c(j)}$$

where $H^{-1}_{(i,j)}$ is the (i,j) entry in the inverse of H. Notice that if U was additively separable as before, then H^{-1} is a diagonal matrix where the (i,i) entry is the reciprocal of the second derivative with respect to good i of the utility function. This implies, $\forall i,\beta(i,i)=\kappa(i)$ and $\forall j\neq i,\beta(i,j)=0$.

The general case of Proposition 1 is as follows:

[16]If H is an infinite matrix, additional regularly assumptions may be necessary. In that case, by inverse we mean the left-hand reciprocal of H, as described in Cooke (2014).

Proposition 6 *Whenever* $\lambda_{00} \in (0,1)$,

$$\varepsilon_T = \int_\Omega (\rho(i)-1)\frac{\frac{X_{00}(i)}{X_{00}}\frac{X_{01}(i)}{X_{01}}}{\frac{X_0(i)}{I_0}}di - \int_\Omega\int_\Omega \beta(i,j)\frac{\lambda_{01}(i)}{\lambda_{01}}\frac{X_{00}(j)}{X_{00}}didj$$
$$+\int_\Omega\int_\Omega \beta(i,j)\frac{X_{00}(j)}{X_{00}}didj\frac{1+\int_\Omega\int_\Omega \beta(i,j)\frac{X_0(j)}{X_0(i)}\frac{X_{01}(i)}{X_{01}}didj}{\int_\Omega\int_\Omega \beta(i,j)\frac{X_0(j)}{I_0}didj}$$

Proof *By the definition of* λ_{00}*:*

$$\lambda_{00}I_0 = \int_\Omega \lambda_{00}(i)p(i)c(i)di$$

Differentiating with respect to $\log(\tau)$ *and using the definition of* ε_T *implies:*

$$\begin{aligned}\varepsilon_T &= -\frac{\int_\Omega\left(\frac{\partial\log(\lambda_{00}(i))}{\partial\log(\tau)}+\frac{\partial\log(p(i))}{\partial\log(\tau)}+\frac{\partial\log(c(i))}{\partial\log(\tau)}\right)X_{00}(i)di}{\lambda_{01}X_{00}\left(1+\frac{\partial\log(w_1)}{\partial\log(\tau)}\right)}\\ &= \int_\Omega(\rho(i)-1)\frac{\frac{X_{00}(i)}{X_{00}}\frac{X_{01}(i)}{X_{01}}}{\frac{X_0(i)}{I_0}}di - \int_\Omega\frac{\frac{\partial\log(c(i))}{\partial\log(\tau)}}{1+\frac{\partial\log(w_1)}{\partial\log(\tau)}}\frac{X_{00}(i)}{\lambda_{01}X_{00}}di\end{aligned}$$

Note that first order conditions for the household are:

$$\frac{\partial U}{\partial c(i)} = \mu_0 p(i)$$

The left hand side may depend on goods other than good i*. Therefore, when we differentiate these first order conditions with respect to* $\log(\tau)$ *we apply the chain rule to get:*

$$\int_\Omega \frac{\partial c(j)}{\partial\log(\tau)}\frac{\partial^2 U}{\partial c(i)\partial c(j)}dj = \mu_0 p(i)\left(\frac{\partial\log(\mu_0)}{\partial\log(\tau)}+\frac{\partial\log(p(i))}{\partial\log(\tau)}\right)$$

For H *as defined above, this can be used to solve for changes in consumption as follows:*

$$\frac{\partial\log(c(j))}{\partial\log(\tau)} = \int_\Omega \beta(i,j)\frac{\partial\log(p(i))}{\partial\log(\tau)}di + \frac{\partial\log(\mu_0)}{\partial\log(\tau)}\int_\Omega\beta(i,j)di$$

Differentiating the budget constraint of the household with respect to $\log(\tau)$ *al-*

lows us to solve for changes in μ_0*:*

$$\frac{\partial \log(\mu_0)}{\partial \log(\tau)} = -\left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau)}\right) \frac{1 + \int_\Omega \int_\Omega \beta(i,j)\lambda_{01}(i)X_0(j)didj}{\int_\Omega \int_\Omega \beta(i,j)X_0(j)didj}$$

Substituting the derivative of μ_0 *into the derivative of* $c(j)$ *and substituting that into the formula for* ε_T *yields the result.*

Note that Propositions 2 and 4 go through in this environment with no changes.

Appendix C: Proof of Proposition 1

In the appendix we write τ_{01} as τ to save notation. Rewriting the definition of λ_{00} yields:

$$\lambda_{00}I_0 = \int_\Omega \lambda_{00}(i)p_0(i)c_0(i)di$$

Noting that the first order condition is:

$$u'(c(i), i) = p(i)\mu_0 + \nu_0(i)$$

Here μ_0 is the Lagrange multiplier on the country 0 household budget constraint and $\nu_0(i)$ is the Lagrange multiplier on the non-negativity constraint for good i. Note that:

$$\nu_0(i) = \begin{cases} 0 & \text{if } c(i) > 0 \\ u'(0,i) - p(i)\mu_0 & \text{if } c(i) = 0 \end{cases}$$

Then:

$$\frac{\partial \nu_0(i)}{\partial \log(\tau)} = \begin{cases} 0 & \text{if } c(i) > 0 \\ -p(i)\mu_0 \left[\frac{\partial \log(p(i))}{\partial \log(\tau)} + \frac{\partial \log(\mu_0)}{\partial \log(\tau)}\right] & \text{if } c(i) = 0 \end{cases}$$

Therefore we can solve for changes in consumption as:

$$u''(c(i), i)\frac{\partial c(i)}{\partial \log(\tau)} = \begin{cases} u'(c(i), i)\left[\frac{\partial \log(p(i))}{\partial \log(\tau)} + \frac{\partial \log(\mu_0)}{\partial \log(\tau)}\right] & \text{if } c(i) > 0 \\ 0 & \text{if } c(i) = 0 \end{cases}$$

Then using the notation from Section 3, we can write this as:

$$\frac{\partial \log(c(i))}{\partial \log(\tau)} = \begin{cases} \kappa(i)\left[\frac{\partial \log(p(i))}{\partial \log(\tau)} + \frac{\partial \log(\mu_0)}{\partial \log(\tau)}\right] & \text{if } c(i) > 0 \\ 0 & \text{if } c(i) = 0 \end{cases}$$

Now we can differentate the definition of λ_{00} and get:

$$\frac{\partial \log(\lambda_{00})}{\partial \log(\tau)} = \int_{\Omega} \left[\frac{\partial \log(\lambda_{00}(i))}{\partial \log(\tau)} + (1+\kappa(i)) \frac{\partial \log(p_0(i))}{\partial \log(\tau)} + \kappa(i) \frac{\partial \log(\mu_0)}{\partial \log(\tau)} \right] \frac{X_{00}(i)}{X_{00}} di$$

Then the budget constraint of the household implies:

$$\frac{\partial \log(\mu_0)}{\partial \log(\tau)} = -\frac{\int_{\Omega} \frac{\partial \log(p_0(i))}{\partial \log(\tau)} (1+\kappa(i)) X_0(i) di}{\int_{\Omega} \kappa(i) X_0(i) di}$$

Substituting in the change in μ_0 term implies:

$$\begin{aligned} \frac{\partial \log(\lambda_{00})}{\partial \log(\tau)} = & \int_{\Omega} \left[\tfrac{\partial \log(\lambda_{00}(i))}{\partial \log(\tau)} + (1+\kappa(i)) \tfrac{\partial \log(p_0(i))}{\partial \log(\tau)} \right] \tfrac{X_{00}(i)}{X_{00}} di \\ & - \int_{\Omega} \kappa(i) \tfrac{X_{00}(i)}{X_{00}} di \tfrac{\int_{\Omega} \frac{\partial \log(p_0(i))}{\partial \log(\tau)} (1+\kappa(i)) X_0(i) di}{\int_{\Omega} \kappa(i) X_0(i) di} \end{aligned}$$

Recall that the trade elasticity can be rewritten as:

$$\varepsilon_T = \frac{\frac{\partial \log\left(\frac{1-\lambda_{00}}{\lambda_{00}}\right)}{\partial \log(\tau)}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau)}} = -\frac{1}{1-\lambda_{00}} \frac{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau)}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau)}}$$

Then using the results above and Lemma 3 yield:

$$\varepsilon_T = \int_{\Omega} \left[\rho(i) - (1+\kappa(i)) \right] \frac{\lambda_{01}(i)}{\lambda_{01}} \frac{X_{00}(i)}{X_{00}} di + \int_{\Omega} (1+\kappa(i)) \frac{X_{01}(i)}{\lambda_{01}} di \frac{\int_{\Omega} \kappa(i) \frac{X_{00}(i)}{X_{00}} di}{\int_{\Omega} \kappa(i) X_0(i) di}$$

This is equivalent to the result.

Appendix D: Proof of Proposition 2

First note that $\lambda_{00} + \lambda_{01} = 1$, so that

$$\frac{\partial \lambda_{00}}{\partial \log(\tau_{01})} = -\frac{\partial \lambda_{01}}{\partial \log(\tau_{01})} \implies \lambda_{00} \frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})} = -\lambda_{01} \frac{\partial \log(\lambda_{01})}{\partial \log(\tau_{01})}$$

Then we can rewrite the definition of the trade elasticity:

$$\varepsilon_T = \frac{\frac{\partial \log(\lambda_{01}/\lambda_{00})}{\partial \log(\tau_{01})}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}} = \frac{\frac{\partial \log(\lambda_{01})}{\partial \log(\tau_{01})} - \frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}} = -\frac{1}{1-\lambda_{00}} \frac{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}}$$

Using the envelope theorem on the dual of the consumer's problem written above yields:

$$\frac{\partial \log(I)}{\partial \log(\tau_{01})} = \int_{\Omega} \frac{\partial \log(p(i))}{\partial \log(\tau_{01})} \frac{p(i)c(i)}{I_0} di$$

Lemma 7

$$\frac{\partial \log(p(i))}{\partial \log(\tau_{01})} = \left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}\right) \lambda_{01}(i)$$

The proof of the lemma follows immediately from the envelope theorem applied to the final good producing firm's problem, and the assumption that intermediate good prices are linear in wages. Using the lemma implies:

$$\frac{\partial \log(I)}{\partial \log(\tau_{01})} = \left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}\right) \int_{\Omega} \frac{X_{01}(i)}{I_0} di = \left(1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}\right) (1 - \lambda_{00})$$

$$\frac{1}{\varepsilon_W} = -\frac{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}{\frac{\partial \log(I)}{\partial \log(\tau_{01})}} = -\frac{1}{1-\lambda_{00}} \frac{\frac{\partial \log(\lambda_{00})}{\partial \log(\tau_{01})}}{1 + \frac{\partial \log(w_1)}{\partial \log(\tau_{01})}} = \varepsilon_T$$

This completes the proof.

Appendix E: Proof of Proposition 3

Note that $\forall \tau, \lambda_{00} \in (0,1) \implies \log(\lambda_{00}) < 0$, and clearly $\varepsilon_T(\tau)^2 > 0$. Therefore, for all τ,

$$sign\left(-\frac{\log(\lambda_{00})}{\varepsilon_T(\tau)^2} \frac{\partial \varepsilon_T}{\partial \log(\tau)}\right) = sign\left(\frac{\partial \varepsilon_T}{\partial \log(\tau)}\right)$$

Suppose that ε_T is increasing in τ. Then the term within the integral is positive for all τ, hence:

$$0 < \int_{\tau_{01}}^{\tau_{AUT}} -\frac{\log(\lambda_{00})}{\varepsilon_T(\tau)^2} \frac{\partial \varepsilon_T}{\partial \log(\tau)} d\log(\tau) = \log\left(\frac{W^{TRADE}}{W^{AUT}}\right) - \frac{1}{\varepsilon_T(\tau_{01})} \log(\lambda_{00})$$

The first term on the right hand side is the gains from trade in the model with a variable trade elasticity, and the second term is the gains from trade in the constant elasticity model. Therefore, the gains from trade are higher in the variable elasticity model than in the constant elasticity model. The same argument applies for the case with a decreasing trade elasticity.

Appendix F: Proof of Proposition 5

Suppose a set of sectors S all have the minimum value of θ such that $\forall i \in S : \theta(i) = \theta^{MIN}$. The sector-level trade elasticity is $-\theta(i)$, so as trade costs get large, low $\theta(i)$ sectors become a larger share of aggregate imports. In the limit, imports from the sectors in S approach one hundred percent of total imports. That is,

$$\lim_{\tau_{01}\to\infty}\sum_{i\in S}\frac{X_{01}(i)}{X_{01}}=1 \quad \text{and} \quad \lim_{\tau_{01}\to\infty}\sum_{i\notin S}\frac{X_{01}(i)}{X_{01}}=0$$

Likewise,

$$\forall i,\ \lim_{\tau_{01}\to\infty}X_{01}(i)=0 \implies \forall i\ \lim_{\tau_{01}\to\infty}\frac{X_{00}(i)}{X_0(i)}=\lim_{\tau_{01}\to\infty}\frac{X_{00}}{I_0}=1$$

Therefore,

$$i\notin S \implies \lim_{\tau_{01}\to\infty}\frac{\frac{X_{01}(i)}{X_{01}}\frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}}=0 \quad \text{and} \quad \forall i,\ \lim_{\tau_{01}\to\infty}\frac{\sum_i\sigma(i)\frac{X_{00}(i)}{X_{00}}}{\sum_i\sigma(i)\frac{X_0(i)}{I_0}}=1$$

Then:

$$\lim_{\tau_{01}\to\infty}\sum_{i=1}^{50}\theta(i)\frac{\frac{X_{01}(i)}{X_{01}}\frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}}=\lim_{\tau_{01}\to\infty}\theta^{MIN}\sum_{i\in S}\frac{\frac{X_{01}(i)}{X_{01}}\frac{X_{00}(i)}{X_0(i)}}{\frac{X_{00}}{I_0}}=\theta^{MIN}$$

and

$$\lim_{\tau_{01}\to\infty}\sum_{i=1}^{50}\left(\sigma(i)-1\right)\frac{X_{01}(i)}{X_{01}}\left[\frac{\frac{X_{00}(i)}{X_{00}}}{\frac{X_0(i)}{I_0}}-\frac{\sum_i\sigma(i)\frac{X_{00}(i)}{X_{00}}}{\sum_i\sigma(i)\frac{X_0(i)}{I_0}}\right]=0$$

Combining these two equations implies the result.